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Econ 812

HW #2 (please type all answers)

1. Suppose 50% of all agents in an economy have U=ln x + ln y, and the other 50% have U=2 ln x + ln y. All agents start with one unit of x and one unit of y. Find the general equilibrium relative prices and allocations.

In the notes we learned that $\frac{p_x}{p_y} = \frac{\sum a_i \overline{y_i}}{\sum \overline{x_i} - \sum a_i \overline{x_i}}$. This can be rewritten as: $\frac{\sum a_i \overline{y_i}}{\sum b_i \overline{x_i}}$.
Suppose for convenience there are 100 agents, and note that we need to normalize the utility functions so that a+b=1.

Then plugging into the above formula: $\frac{p_x}{p_y} = \frac{50*.5*1+50*2/3*1}{50*.5*1+50*1/3*1} = 1.4$. (Notice that the absolute number of agents actually makes no difference for the results). Each individual has 2.4 units of income (1.4*1+1*1). Using the constant income fractions rule, we know that the first type of agent spends 50% of their income on each good, while the second type spends 2/3 on x and 1/3 on y. So:

The first type of agent spends 1.2 units of income on each good, and therefore consumes 1.2/1.4=.857 units of good x, and 1.2/1=1.2 units of good y. In other words, each type 1 agent sells .143. units of good x to get .2 extra units of good y.

The second type of agent spends 2.4*2/3=1.6 units on good x and 2.4*1/3=.8 units on good y. So he consumes 1.6/1.4=1.143 units of good x and .8/1=.8 units of good y. In other words, he buys .143 units of x using .2 units of y.

2. Re-do problem #1, assuming that the first type of agent starts with 2 units of x and 0 of y and the second type of agent starts with 2 units of y and 0 of x.

$$\frac{p_x}{p_y} = \frac{50*.5*0+50*2/3*2}{50*.5*2+50*1/3*0} = \frac{50*2/3*2}{50*.5*2} = 4/3.$$

The first type of agent now has 8/3 units of income; the second type 2 units. The income fractions remain the same. So:

The first type consumes 8/3*.5/(4/3)=1 unit of good x and 8/3*.5/1=4/3 units of good y. They sell 1 x to get 4/3 y.

The second type consumes 2*2/3/(4/3)=1 unit of good x and 2/3/1=2/3 units of good y. They sell 4/3 y to get 1 x.

3. Re-do problem #1, assuming that all agents have U=x+y. (Hint: At disequilibrium prices, agents want to consume only x or only y).

The only possible price ratio is $\frac{p_x}{p_y}=1$. If x were cheaper than y, all agents would want to consume only x; if y were cheaper than x, all agents would want to consume only y. Since all agents start with 1 unit of x and 1 unit of y, the only equilibrium is one where each person consumes exactly 1 x and 1 y.

4. Suppose you can redistribute x, but not y. Returning to problem #1, what exactly must you do to:

(a) make the equilibrium utility of the first type of agents equal to .5,
(b) give all agents of the second type the same utility,
(c) and make type-2 agents' utility as high as possible conditional on (a)?

The natural strategy here is to figure out how much endowment must be taken from each type 2 agent and given to each type 1 agent to make the type 1 utility .5. That will automatically leave all type 2 agents with equal utilities, which will be as high as possible conditional on redistribution.

Let the new type 1 endowment be $\bar{x}$; then the new type 2 endowment will be $2-\bar{x}$. Plugging into the price formula:

$$\frac{p_x}{p_y}=\frac{50*.5*1+50*2/3*1}{50*.5*\bar{x}+50*1/3*\left(2-\bar{x}\right)}=\frac{7}{\bar{x}+4}.$$

Thus, agent 1 agents each have a total income of $\frac{7}{\bar{x}+4}\bar{x}+1$. They will still spend half their income on x and half on y, so they consume $\frac{\frac{7}{\bar{x}+4}\bar{x}+1}{2*\frac{7}{\bar{x}+4}}=\frac{4\bar{x}+2}{7}$ of x and $\frac{\frac{7}{\bar{x}+4}\bar{x}+1}{2}=\frac{4\bar{x}+2}{\bar{x}+4}$ of y. Thus, the utility of the first type of agent is:

ln $\frac{4\bar{x}+2}{7}$ + ln $\frac{4\bar{x}+2}{\bar{x}+4}$. Now just set that equal to .5 and solve:

ln $\frac{4\bar{x}+2}{7}$ + ln $\frac{4\bar{x}+2}{\bar{x}+4}$=.5

$\frac{4\bar{x}+2}{7}*\frac{4\bar{x}+2}{\bar{x}+4}$=e$^{.5}$

$$16\bar{x}^2+4.46\bar{x}-42.16=0$$

Using the quadratic formula and discarding the extraneous solution, $\bar{x} = 1.49$. Thus, it will be necessary to redistribute .49 units of x from each type 2 agent to each type 1 agent.

Plugging in, this implies that $\frac{p_x}{p_y} = \frac{7}{1.49+4} = 1.275.$ The income of the type 1 agents is now 2.90, so they consume 1.45/1.275=1.137 units of x and 1.45/1=1.45 units of y. In other words, then, after redistributing .49 units of x to the type 1 agents, they sell (1.49-1.137)=.353 units of x to buy .45 extra units of y. Double-checking the result, the utility of the type 1 agents is ln 1.137 + ln 1.45 = .5.

This leaves each type 2 agent with .863 units of x and .55 units of y, with a utility level of -.893.

5. (half page) Use general equilibrium analysis to explain why barbers earn higher real wages now than in 1900, even though there has been little technological progress in this industry.

Labor productivity has risen dramatically outside of the barbering industry, leading to increased labor demand. The only way to induce barbers to continue barbering, then, is to increase their real wages enough that they do not want to quit and move to a progressing sector. If wages did not rise in technologically stagnant sectors, eventually no one would want to work in them. In other words, in spite of the lack of technological change in barbering, technological changes in *other* industries have indirectly exerted a major effect on barbering. As the value of time rises, people become more willing to pay others to cut their hair - a increase in demand for barbers. And as non-barbering options have grown more lucrative, the supply of barbers has decreased. The net effect is a large rise in barbers' wages even though barbers as a fraction of the labor force may be about the same as a century ago.

6. (half page) Explain how betting markets could be used to resolve a practical controversy of your choice. Carefully explain the exact bet or bets that would need to be offered. Would you be willing to bet against the market?

I am a big fan of the signaling theory of education, which we'll discuss after the midterm. One of its main implications is that drastically cutting education spending would not actually hurt average real wages. If far fewer people finish college, signaling theory tells us that employers will no longer make such a negative inference about the productivity of workers without college degrees. One possible bet to test my theory is this: I sell promises to pay $1 if per-capita government spending on higher education falls by 2010, AND the fraction of individuals 22-30 years with college degrees falls, AND the average real wages of this cohort do not fall. I also sell promises to pay $1 if per-capita government spending on higher education falls by 2010, AND the fraction of individuals 22-30 years with college degrees falls, BUT the average real wages of this cohort do fall. By comparing the prices of the two securities you could then get a conditional estimate. E.g. if the first security goes for 1 penny, and the second goes for .5 cents, that indicates a conditional probability of 2/3 that I'm right. I would not bet against the market unless if gave me less than a 1-in-3 chance of being correct.