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Econ 812

HW #3 Answer Key (please type all answers)

1. Solve the following game using strict dominance:

	Player 2			
Player 1		Attack	Defend	Retreat
	Attack	1,2	0,5	2,1
	Defend	2,0	**6,3**	3,2
	Retreat	3,4	1,5	4,3

Player 2 is always better off playing Defend. After eliminating Player 2's Attack and Retreat Columns, Player 1 simply picks the biggest payoff out of 0, 6, and 1. The answer, obviously, is 6 for Defend.

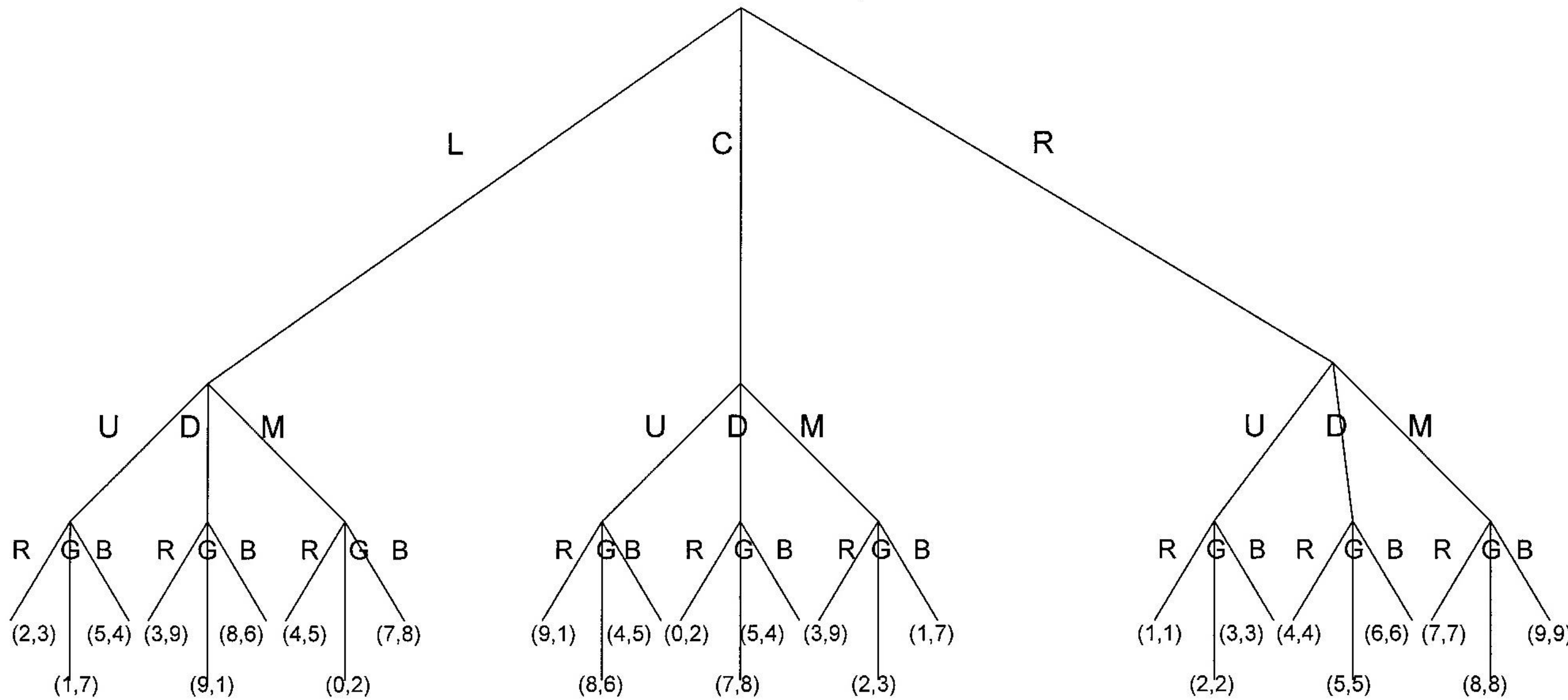
2. Diagram the following game in extensive form, then solve it using backwards induction.

a. Player 1 chooses left, center, or right.
b. Player 2 chooses up, down, or middle.
c. Player 1 chooses red, green, or blue.

Payoffs are as follows:

LUR	2,3	CUR	9,1	RUR	1,1
LUG	1,7	CUG	8,6	RUG	2,2
LUB	5,4	CUB	4,5	RUB	3,3
LDR	3,9	CDR	0,2	RDR	4,4
LDG	9,1	CDG	7,8	RDG	5,5
LDB	8,6	CDB	5,4	RDB	6,6
LMR	4,5	CMR	3,9	RMR	7,7
LMG	0,2	CMG	2,3	RMG	8,8
LMB	7,8	CMB	1,7	RMB	9,9

(See attached diagram). At the final nodes, player 1 chooses LU**B**, LD**G**, LM**B**, CU**R**, CD**G**, CM**R**, RU**B**, RD**B**, and RM**B**. Therefore at the preceding node, player 2 chooses L**M**B, C**M**R, and R**M**B. Player 1 therefore is deciding between payoffs of 7, 3, and 9, and therefore picks **R**MB.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(2,3) (5,4) (3,9) (8,6) (4,5) (7,8)
(1,7) (9,1) (0,2)
(9,1) (4,5) (0,2) (5,4) (3,9) (1,7)
(8,6) (7,8) (2,3)
(1,1) (3,3) (4,4) (6,6) (7,7) (9,9)
(2,2) (5,5) (8,8)

Problem 2

3. Find the PSNE.

		Player 2	
Player 1		Left	Right
	Up	**9,9**	9,9
	Down	0,0	**10,9**

Up, Left is one PSNE: Player 1 would drop from 9 to 0 if he switched, and Player 2 would go from 9 to 9. Down, Right is the other PSNE: Player 1 goes from 10 to 9 if he switched, and Player 2 goes from 9 to 0 if he switches. Both players would want to change their behavior if they were at Down, Left, and Player 1 would want to change if they were at Up, Right.

4. Find the MSNE for problem 3.

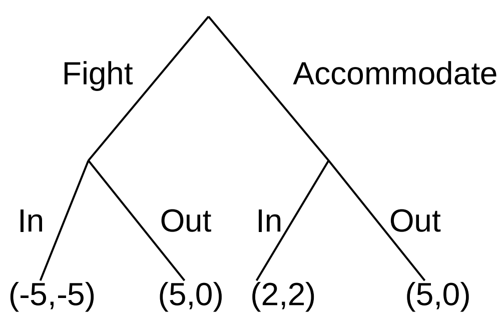
Designate Player 1's probability of playing Up as σ and Player 2's probability of playing Left as ϕ. Then Player 2 is indifferent between Left and Right if:

$9\sigma+0*(1-\sigma)=9\sigma+9*(1-\sigma)$. This only holds if $\sigma=1$; i.e., player 1 players Up with certainty.

Similarly, Player 1 is indifferent between Up and Down if:

$9\phi+9*(1-\phi)=0*\phi+10*(1-\phi)$. This only holds if $\phi=1/10$.

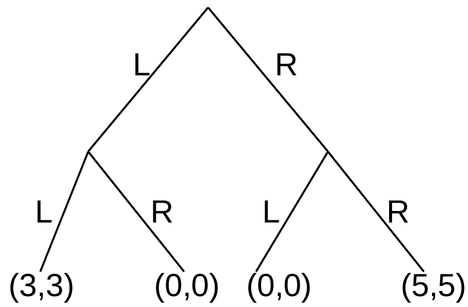
5. Re-diagram the extensive form of Entry game if the Incumbent moves first instead of the Entrant. Does the subgame perfect equilibrium change? Explain.



Yes, the SGPNE does change. Using backwards induction, we see that the Entrant will never play In if the Incumbent first plays Fight, though it will play In if the Incumbent players Accommodate. Therefore when it decides between Fight and Accommodate, the Incumbent is choosing between (**Fight**, Out) and (**Accommodate**, In), with respective payoffs of 5 and 2. Therefore it chooses Fight, and (Fight, Out) becomes the SGPNE.

Changing the order of play drastically changes this game. Since the Incumbent commits to a strategy BEFORE the Entrant does, its threat to Fight is "locked-in" and therefore credible. In other words, the Incumbent in this variant of the game pre-commits to Fight, which scares off the Entrant.

6. Re-diagram the extensive form of the Coordination game if Player 1 moves first. Does the set of equilibria (pure strategy and mixed strategy) change? Why or why not?

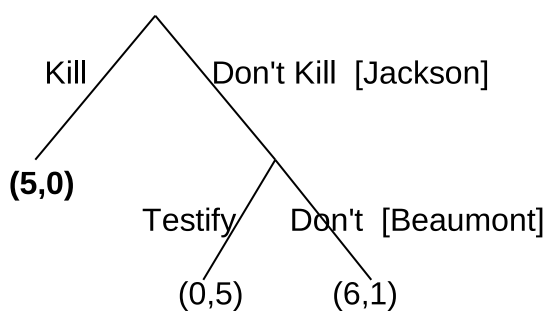


If we do not impose subgame perfection, the PSNE stay the same. Player 1 will play L if he knows that Player 2 will play L. However, the MSNE disappears, because once Player 1 moves, Player 2 is always better off playing either 100% L or 100% R.

If we do impose subgame perfection, though, the only equilibrium is now R,R. Using backwards induction, Player 2 plays (L,**L**) or (R,**R**). Therefore player 1 is choosing between 3 and 5, and gets the latter by playing (**R**,R).

7. Explain a simple, original, real-world strategic situation of your choice. Draw its extensive form and normal form. What are the pure and mixed strategy Nash equilibria? (half page)

In the movie *Jackie Brown*, Samuel L. Jackson plays a drug dealer who kills one of his employees, Beaumont. The motive, explains Jackson, was that Beaumont had been caught dealing drugs and therefore, if he stayed alive, would strike a deal to testify against Jackson. Nevertheless, Jackson would prefer not to kill Beaumont if Beaumont did not testify, if only to avoid the murder charge. So if we were modeling this situation, it might look like this:



		Beaumont	
		Test.	Don't
Jackson	Kill	**5,0**	5,0
	Don't	0,5	6,1

There is one PS Nash equilibria: (Kill, Testify). Backwards induction shows that this NE is also subgame perfect.

There is also a MSNE of a strange kind. Jackson must play Kill with p=1 to keep Beaumont indifferent. But Beaumont can play Testify with any $p \geq 1/6$. (If $p=1/6$, Jackson is indifferent between Kill and Don't; if $p>1/6$, Jackson strictly prefers Kill). Note, however, that the MSNE is not subgame perfect, because Beaumont would always Testify if Jackson does not play Kill.

8. Use game theory to analyze strategy in a sporting event of your choice. Pay particular attention to mixed strategy equilibria. (half page)

Consider volleyball. For one thing, you want to deliver a variety of serves. If you give all hard or all easy serves, the opposing team will move players into the zone where you keep serving. Similarly, you want to vary the part of the court you're aiming for. You can see mixed strategy behavior in later stages of the game as well. Even if one player is clearly the best, you will not want to give him in the ball on every serve, because the opposing team will focus all of its resources on him. Instead, you try to randomize to make your team's efforts harder to counteract.