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Econ 812

HW #4 Answer Key (please type all answers)

1. Suppose two players play the following games, in order:

Game #1:

		Player 2	
Player 1		Coop	Don't
	Coop	5,5	1,6
	Don't	6,1	2,2

Game #2:

		Player 2	
Player 1		Left	Right
	Left	5,1	0,0
	Right	0,0	1,5

Does playing Game #2 make cooperation in Game #1 sustainable if players do not discount future payoffs (β=1)? Why or why not?

The strictly dominant strategy of Game #1 is of course (Don't, Don't); Game #2 has two pure strategy equilibria: (Left, Left) and (Right, Right). (Coop, Coop) can be sustained in the following way if the players have access to a jointly viewable random number generator after Game #1:

If (Coop, Coop) or (Don't, Don't), play (Left, Left) with probability .5 (e.g., if the random number generator yields a number below its median), and (Right, Right) with probability .5 (e.g. if the random number generator yields a number above its median).

If (Don't, Coop) play (Right, Right).

If (Coop, Don't) play (Left, Left).

Given this strategy, if a player cooperates in turn one, he earns (5 + .5*5 + .5*1)=8, but only gets (6+1)=7 if he defects in turn one.

Without a random number generator (or if you view the random number *before* game 1!), it will be impossible to sustain (Coop, Coop), because one player will be better off if the game 2 equilibrium switches, making that player impossible to punish. E.g., if the players play (Left, Left) if (Coop, Coop), (Coop,Don't), or (Don't,Don't) but (Right, Right) if (Don't, Coop), player 1 gets 10>7 if he Cooperates, but player 2 gets 6<7 if he plays Don't.

Note however that you could sustain the equilibrium (Left, Left), (Coop, Don't) under the following conditions:

Both players play Left if player 1 played Coop, and both play Right if player 1 played Don't. Then Player 1 gets (1+5)>(2+1), and Player 2 gets (6+1)>(5+1). (You could of course flip the roles of Player 1 and Player 2).

If a cooperative equilibrium is possible, would you expect it to be likely? Why or why not?

None of the cooperative strategies seem too likely to occur in practice without extensive communication due to their complexity. None of them seem particularly focal either, at least to me.

2. Suppose you are playing an infinitely-repeated PD game with payoffs from the Week 3-4 notes. The game ends each turn with probability p, and players discount the future by β. Under what conditions can *trigger strategies* sustain cooperation?

		Player 2	
Player 1		Coop	Don't
	Coop	5,5	0,6
	Don't	6,0	**1,1**

If a player cooperates, he earns: $\sum_{t=0}^{\infty} \beta^t (1-p)^t 5$.

If he defects, he earns: 6 + $\sum_{t=1}^{\infty} \beta^t (1-p)^t$

Now note that $\beta^t (1-p)^t = (\beta(1-p))^t$. Then these expressions reduce to:

$$\frac{5}{1-\beta(1-p)} \geq 6 + \frac{\beta(1-p)}{1-\beta(1-p)}$$

This inequality is satisfied so long as:

$$5 + \frac{4\beta(1-p)}{1-\beta(1-p)} \geq 6$$

which reduces to:

$$\beta(1-p) \geq 1/5.$$

3. In an infinitely-repeated reputation game (with the payoffs from the Week 5 notes), imagine customers only punish a turn of Cheat with 50% probability of not buying the next turn. Solve for the seller's critical value of β.

The payoffs are:

	Buy	Don't
Cheat	10,-2	0,0
Don't	5,2	0,0

Now if the seller is honest, he still gets a payoff of 5 every turn.

If he cheats, though, he risks only a 50% chance of getting 0 on the next turn, and in any case still earns 5 after the punishment turn. So he plays Don't only if:

$$\frac{5}{1-\beta} \geq 10 + .5*0\beta + .5*5\beta + \sum_{t=2}^{\infty} 5\beta^t \ .$$

The infinite sums on the left and right hand sides cancel, yielding:

$$5 + 5\beta \geq 10 + 2.5\beta$$

which is satisfied only if $\beta \geq 2$. In other words, the seller will definitely cheat unless he cares MORE about the future than the present.

Some students have however pointed out a better deviation: rather than cheating once, the seller could cheat always. This would be profitable unless:

$$\frac{5}{1-\beta} \geq 10 + .5\sum_{t=1}^{\infty} 10\beta^t$$

Cancelling infinite sums on the left and right hand sides yields:

$$5 \geq 10$$

so deviation is always profitable, and cooperation cannot be sustained with this weak punishment.

4. Analyze the expected efficiency properties of the MSNE where two firms simultaneously incur sunk costs. Use the notation from the notes (V.I.) for the firms; designate maximum consumers surplus as CS_{max}, consumers surplus under monopoly as CS_{mon}, and note that if a good is not produced then consumers surplus is 0. Under what conditions is contestability with simultaneous sunk costs welfare-dominated by simple monopoly?

In the MSNE, each firm plays In with probability $\frac{\Pi^m}{\Pi^m + a}$. So there are three cases to consider, and they occur with the following probabilities:

Number of Firms that Play In	Probability
2	$\left(\frac{\Pi^m}{\Pi^m + a}\right)^2$
1	$2\left(\frac{\Pi^m}{\Pi^m + a}\right)\left(\frac{a}{\Pi^m + a}\right)$
0	$\left(\frac{a}{\Pi^m + a}\right)^2$

To get total surplus, we just add producer plus consumer surplus:

Number of Firms that Play In	Producer Surplus	Consumer Surplus	Total Surplus
2	-2a	CS_{max}	-2a + CS_{max}
1	Π^m	CS_{mon}	Π^m + CS_{mon}
0	0	0	0

Under monopoly, total surplus is (Π^m+ CS_{mon}) with probability 1. So when is contestability welfare-dominated by simple monopoly? When the expected surplus under contestability is less than (Π^m + CS_{mon}):

$$\left(\frac{\Pi^m}{\Pi^m + a}\right)^2 (-2a + CS_{max}) + 2\left(\frac{\Pi^m}{\Pi^m + a}\right)\left(\frac{a}{\Pi^m + a}\right) (\Pi^m + CS_{mon}) < (\Pi^m + CS_{mon})$$

Thus, suppose that Π^m=a. Then contestability is welfare-dominated if:

$$.25^* (-2a + CS_{max}) + .5^* (a + CS_{mon}) < (a + CS_{mon})$$

which reduces to:

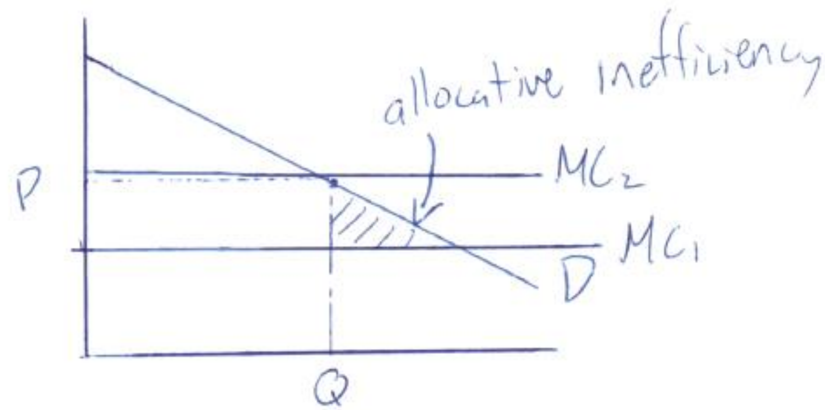
$$CS_{max} < 2CS_{mon} + 4a$$

i.e., monopoly welfare dominates if the consumer surplus under competition is less than twice the consumer surplus under monopoly plus four times the fixed cost.

5. How does cost heterogeneity affect the welfare equivalence of perfect competition and Bertrand oligopoly? How much is this likely to matter in the real world? Use diagrams to illustrate your answer. (one paragraph)

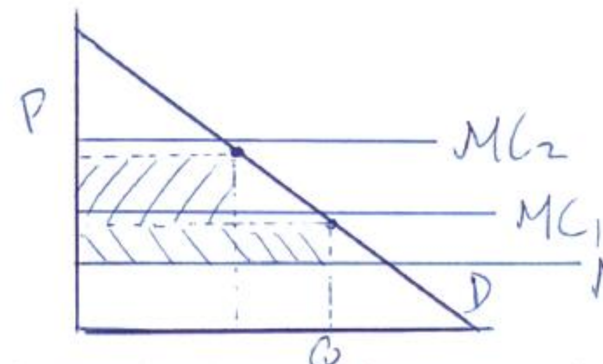
With heterogeneous costs, Bertrand oligopoly will fall short of the competitive ideal. The most efficient firm will charge a price just under the MC of the second most-efficient firm, and therefore prices in excess of its own MC. Some allocative inefficiency therefore exists under Bertrand oligopoly unless the two most efficient firms are equally efficient.

In the real world, though, this is probably a small problem. The ability gap between the best and the second-best firm is likely to be modest.



6. Analyze the incentive for firms to increase their productive efficiency under Bertrand oligopoly. Use diagrams to illustrate your answer. (one paragraph)

Under Bertrand oligopoly, the most efficient firm gets to earn monopoly profits. This gives firms an incentive to invest in (excludable) R&D in order to figure out how to cut costs. The most efficient firm can earn more profits if it increases the gap between itself and the second best firm; firms who are not the best can earn monopoly profits if they become the best.



If firm 1 cuts costs from MC_1 to MC_3, it gets $(MC_1 - MC_3)$ in extra profits on each unit. If firm 2 cuts costs from MC_2 to MC_3, it gets the entire area marked [hatched box].

7. Suppose that there are 4 Cournot competitors with MC=0 and no fixed costs. Prove that at least one of these firms would like to split into two firms; i.e., that $2^*\Pi(5)>\Pi(4)$.

With four firms, $Q=\frac{4a}{5b}$, and profits=revenue=$\frac{PQ}{4}=\left[a-b\frac{4a}{5b}\right]\frac{a}{5b}$, which simplifies to:

$\left[\frac{1}{5}a\right]\frac{a}{5b}=\frac{a^2}{25b}$.

With five firms, similarly, the profit of one firm is:

profits=revenue=$\frac{PQ}{5}=\left[a-b\frac{5a}{6b}\right]\frac{a}{6b}$, which simplifies to:

$\frac{a^2}{36b}$. But a firm that splits gets the profits for TWO firms, so we need merely verify that:

$$2*\frac{a^2}{36b} > \frac{a^2}{25b}$$, which it clearly does.

(If you want a challenge, generalize this result to show that firms want to split so long as $N > \sqrt{2}$, which in plain English means that firms will want to split unless you begin with a monopoly).

8. Suppose Cournot firms have a fixed cost, K, but 0 MC. If P=20-Q, solve for the free entry value of N as a function of K. Briefly explain why the first-best outcome sets N=1.

From the last problem, we have seen that with zero marginal costs Cournot revenue as a function of N is:

$$\frac{a^2}{(N+1)^2}$$

Under free entry, firms will enter until TR=TC.

Since P=20-Q, a=20.

Using these facts:

$$\frac{400}{(N+1)^2} = K$$

which simplifies to:

$$N = \frac{20}{\sqrt{K}} - 1.$$

(Of course, in a discontinuous world, we would look for the biggest N such that $N < \frac{20}{\sqrt{K}} - 1$).

Thus, the bigger the fixed costs, the smaller the equilibrium number of firms. The first-best outcome, however, would set N=1, Q=20, and P=0. Given the cost function, the entry of each firm beyond the first leads to a waste of K; one firm could produce the total output of N firms at a total cost of K(N-1) less.