Extreme Walrasian Dynamics: The Gale Example in the Lab

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Abstract

We study the classic Gale economy in a series of laboratory markets. Walrasian dynamics predict prices will diverge from an unstable, equitable interior equilibrium towards infinity or zero depending only on initial prices. The stable, inequitable equilibria selected by these dynamics give all of the gains from exchange to one side of the market. We argue this economy provides a strong robustness test of previous work demonstrating the predictive and explanatory power of Walrasian dynamics in laboratory markets. Our results show surprisingly strong support for these predictions. In most sessions one side of the market eventually gains more than 20 times the other side through exchange, leaving the disadvantaged side to trade for mere pennies. We also find evidence that these dynamics are sticky, resisting exogenous interventions attempting to reverse their trajectories.

Keywords: Tatonnement, Disequilibrium, General Equilibrium, Experiment

JEL codes: C92, D50

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"[F]or the case of two goods, one always has global stability ... Nevertheless, some queer things can happen even in this case." —David Gale (1963)

1. Introduction

General equilibrium theory is a cornerstone of modern economics and our core account of the nature of competitive markets. However, the theory has usually been focused more on the existence and character of competitive equilibrium than on how, when and why economies come to be in equilibrium. Given the computational and epistemic requirements for calculating a competitive equilibrium, it seems implausible that economic agents could ever "think" their way there. More likely adaptive dynamic processes govern disequilibrium prices, guiding them towards or away from equilibria. Until and unless we understand these dynamic processes, it is hard to assess general equilibrium theory's usefulness for predicting and explaining the behavior of competitive markets.

On this front there has been no shortage of theory. Accounts of disequilibrium dynamics stretch back to Walras (1877), and the quest for a satisfying theory was an active pursuit until the 1970s (e.g. Hahn and Negishi (1962), Uzawa (1962), Hurwicz et al. (1975)) when it died off arguably for want of empirical nourishment.[1] Modern observers have wondered whether existing theories of dynamics are empirically meaningful given that they are typically founded on some variation of tatonnement, a centralized price adjustment mechanism that differs substantially from most naturally occurring markets. In a wide-ranging survey, Duffie and Sonnenschein (1989) conclude that because actual market prices are not determined by the tatonnement mechanism, "few would argue today that it is a useful way to select from Walrasian equilibria."

Recently, laboratory research has stepped in to fill the empirical gap (e.g., Plott (2000), Plott (2001), Anderson et al. (2004), Hirota et al. (2005), Gjerstad (2007)), and findings have in fact been broadly supportive of Walras' hypothesis that price dynamics

[1] A fact general equilibrium theorists sometimes lament (see for example Kirman (1989).

are intimately related to and driven by a market's excess demand.[2] Moreover, this literature has suggested that Walrasian notions of dynamics and stability have predictive power even in distinctly non-tatonnement market institutions (virtually all of this literature uses the double auction institution).[3]

In this paper we provide perhaps the strongest test to date of the Walrasian hypothesis by experimentally studying a simple economy in which Walrasian dynamics predict highly implausible outcomes. In the Gale (1963) 2-good economy, Walrasian dynamics push disequilibrium prices of the non-numeraire good *away* from an equitable (but unstable) interior competitive equilibrium towards infinity or zero. Disequilibrium price paths eventually induce agents on one side of the market to give goods away for free, along with all gains from trade, within one of a pair of corner equilibrium sets. Remarkably, which side of the market gives away its goods depends not on structural parameters of the economy but purely on the market's initial price and the dynamics it sets off. Here, in Balogh and Streeten's (1951 p. 75) memorable phrase "the invisible hand does its work by strangulation."

Gale's economy provides a powerful stress test of the Walrasian hypothesis for two reasons. First, the Walrasian predictions in this economy have been dismissed as implausible by leading theorists,[4] whereas interior equilibration or global instability are highly appealing alternative hypotheses. Indeed, the economy is often used as a *reductio ad absurdum*, a cautionary tale concerning the limits of aprioristic reasoning about markets. Chipman (1965) writes of the Gale example, "It is best to consider it as a sobering reminder that the pure theory admits of many strange possibilities that cannot be

[2] Marshallian dynamics, where quantities adjust given differences between buyer and seller prices, have been found to be more successful in economies with negative externalities (see Plott and George, 1992), and positive externalities, (Plott and Smith, 1999) and recently in an economy with continuous probabilistic market entry (Alton and Plott, 2009).

[3] Although modern theorists frequently motivate tatonnement by describing a fictional centralized mechanism, Walras himself did not use such a mechanism to motivate his theory. In fact Walras conceived of tatonnement as a theory regarding the process governing decentralized markets (see Walker (1996)). Thus Walras would likely have been less surprised than modern observers to learn that tatonnement does a good job of anticipating the behavior of decentralized laboratory markets. We thank Omar Al-Ubaydli for pointing this out to us.

[4] Chipman (1965, p. 730) concludes that Arrow and Debreu (1954) rule out Gale-like corner outcomes from consideration as equilibria at all, so that the example's limiting paths "do not qualify as equilibrium solutions, and the [Gale] example becomes one of global instability."

ruled out by a priori reasoning." It is precisely the fact the Gale example is transparently "strange" (Chipman, 1965), "queer" (Gale, 1963) and implausible (Arrow and Debreu, 1953) that makes it a limiting robustness test for Walrasian dynamics. If Walrasian predictions work here it seems they will work nearly anywhere.[5]

Second, the Gale example (unlike the similarly famous and previously studied Scarf example) is a two good economy that can be easily implemented in a single commodity double auction with an equal number of net buyers and net sellers. Price dynamics and their implications for each side of the market are utterly transparent, as the trader need only pay attention to one price series to quickly understand the character of dynamics. One side of the market has both sufficient information and powerful incentives to resist Walrasian price trajectories.

This project began as a friendly argument among coauthors concerning the robustness of Anderson et al. (2004), which identified price cycles across periods in a laboratory implementation of Scarf's example. A reasonable inference from this research is that, where they conflict, Walrasian dynamics are more important predictors of outcomes than the fixed points which lie at the heart of equilibrium economics. Two authors conjectured that given enough experience (here 13 or more trading periods per session, nearly double the number observed in Anderson et al.), large enough markets (10 subjects of each type, double the number per type in Anderson et al.), transparent enough dynamics, and contemptible enough outcomes (under Gale, half of the subjects' earnings are devastated by dynamic trajectories), Walrasian dynamics would fall apart and criteria other than tatonnement stability would come to govern behavior, leading economies to eventually converge to the interior competitive equilibrium.

The skeptical authors were proved wrong. We report robust evidence that prices in laboratory Gale economies resist the interior competitive equilibrium and march upwards or downwards towards the corner equilibria. In fact, prices became as high or as low as one could expect given the discreteness of the space of goods in the lab economy, so that subjects on the "wrong" side of the market were left trading their entire allotment

[5] As previously mentioned, one notable exception is economies with strong externalities, where Marshallian dynamics seem to be dominant (see Plott and George, 1992).

of goods for a few pennies. We also discover that dynamics, once seeded, are sticky and difficult to reverse.

A handful of earlier studies have shown emergent prices in partial equilibrium environments that disadvantage one side of the market (e.g. Holt et al. (1986)). What makes the Gale example curious is that the extreme predictions come about not because of structural factors such as the number of traders, or the basic character of supply and demand. Rather the predictions are driven by something as apparently arbitrary as the economy's initial state and the dynamics the initial state sets off. Moreover, unlike previous studies --in which inequitable outcomes were unique equilibria-- the Gale example has an alternative and equitable equilibrium. It is the inherently dynamic cause of extreme inequity and the existence of a reasonable alternative that makes the example so counterintuitive and the laboratory evidence supporting it compelling.

The remainder of the paper is organized as follows. In section 2 we review Walras' theory of tatonnement, introduce our parameterization of a Gale economy, describe our experimental design, and lay out our primary experimental questions. In section 3 we present results from our experiment. We conclude with a discussion of our results in section 4.

2. The Gale Example: Theory and Experimental Design

2.1 Walrasian Dynamics and the Gale Example

Tatonnement is the earliest and best known of the classical theories of market price dynamics. The process begins with an arbitrary vector of initial prices that induce a corresponding vector of excess demands. If a good is in excess demand its price increases, while its price decreases if the good is in excess supply. This process is iterated indefinitely until excess demand for each good is zero and a competitive equilibrium is reached. It is only in equilibrium that trades are actually executed and endowments adjusted.[6] In our economy there are only two commodities x and y. We normalize the price of x to one, letting p be the price of good y, and express tatonnement adjustment in terms of the time derivative of this price:

$$\dot{p} = f(z(p)) \tag{1}$$

where $z(p)$ is the excess demand for good y given p, and f is a sign preserving function of excess demand. The discrete time analog is a similar difference equation. A finite price $p^* > 0$ is a competitive equilibrium price if $z(p^*) = 0$. A competitive equilibrium price is *locally stable* if there exists $\varepsilon > 0$ such that for all $\delta \neq 0$ where $|\delta| < \varepsilon$, $\delta z(p^* + \delta) < 0$. A competitive equilibrium price is *globally stable* if $\delta z(p^* + \delta) < 0$ for all finite δ. Local and global instability are defined similarly but the $\delta z(\cdot)$ conditions have opposite sign.

We consider the two good (x and y), two agent type (Even and Odd) exchange economy represented by the Edgeworth box on the left hand side of Figure 1. The origin of Even's coordinate axis is the lower-left corner of the Edgeworth box, Odd's the upper-right. Each agent i has preferences

$$U_i(x_i, y_i) = \min\{x_i + a_i, b_i y_i\}. \tag{2}$$

For an Even type agent i, $a_i = -1349$ and $b_i = 35.5$ (approximately), and his endowment is $(x_i, y_i) = (400,15)$. For an Odd type agent j, $a_j = 3947$ and $b_j = 658$, and his endowment is $(x_j, y_j) = (5600,5)$.

[6] Scarf (1960) provided a 3 agent, 3 good economy with a unique competitive equilibrium that is globally unstable under the basic tatonnement adjustment procedure; tatonnement prices converge to a limit cycle about the competitive equilibrium. As previously mentioned, Anderson et al. (2004) report that mean prices across periods track this limit cycle in a laboratory implementation of Scarf's example.

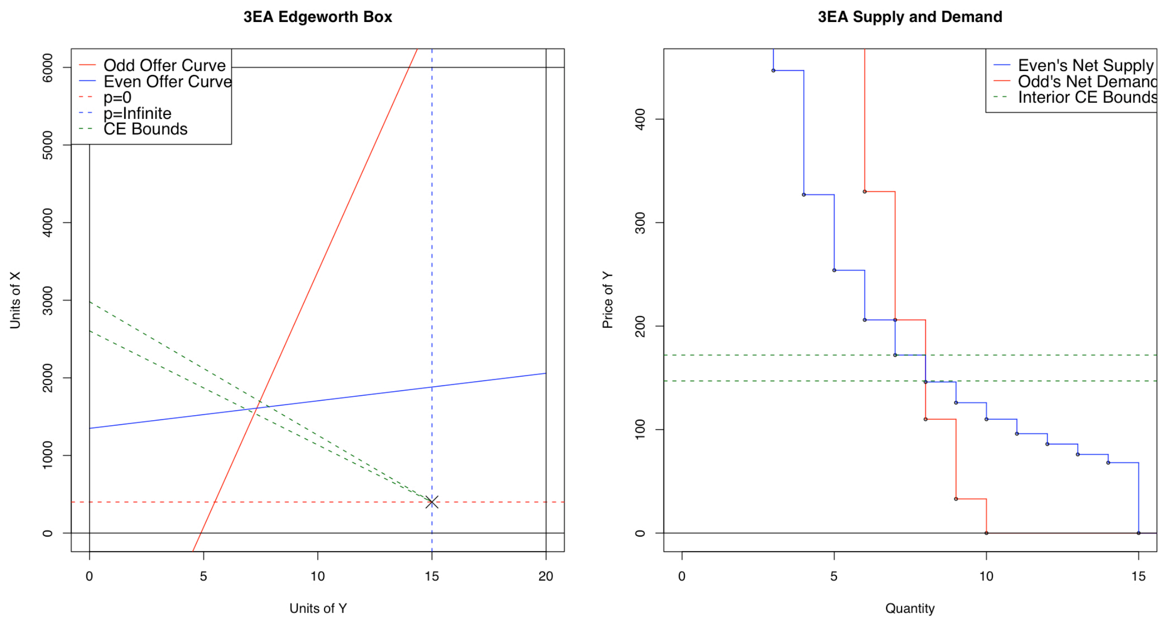

Figure 1: Edgeworth box and net supply and demand functions.

The economy is a variation of one described by Gale (1963).[7] The activity rays of the two agent types[8] intersect in the Edgeworth box at an interior competitive equilibrium (ICE) cleared by a price of (approximately) p=158. This equilibrium is unexceptional in many respects. It is interior, equitable (each party receives identical payoffs and gains from trade under the scaling transformation we adopt), and is defined by an excess demand of zero. However, it is globally unstable under tatonnement. At prices higher than (clockwise from) the ICE price, Odd's demand for good y exceeds Even's supply, generating positive excess demand and by (1) prices will eventually move towards infinity. At prices lower than (counter clockwise from) the ICE price the reverse is true, and by (1) prices will eventually converge to zero.

In the discrete environment we implement in the lab, there is a set of ICE prices contained within the cone $p \in [147,172]$ which emanates from the endowment allocation and is centered on the confluence of the activity rays (this cone is drawn with dashed lines in the Edgeworth box). The right hand side of Figure 1 provides a view of the

[7] Gale considers preferences where $b_i = 0$ for all agents i. As described below, we introduce this non-zero "intercept" term to mitigate a difficulty in implementing discrete Leontief economies in the lab.
[8] The activity ray for agent i is the line segment $x = a_i y - b_i$, $x, y > 0$. It represents the set of strictly positive consumption bundles for which the entire quantity of each good is necessary for i's utility at that bundle, and strictly contains the agents offer curve.

discretized economy, plotting net supply and demand functions with respect to the endowment. Net supply and demand are equal for each discrete price within the ICE cone, resulting in zero excess demand. Clearly demand exceeds supply at prices above the ICE range and supply exceeds demand below, destabilizing the interior equilibrium set.

Walrasian dynamics drive adjustment until prices reach either 0 or infinity[9] and can no longer adjust in the direction of excess demand.[10] These prices are, by convention, competitive equilibria but of an odd form. At each equilibrium, excess demand is far from zero and allocations are non-interior (we call them corner competitive equilibria, or CCE). The equilibria are also highly inequitable; in each case one side of the market gives away some amount of a commodity to the other side for free, along with all of the gains from trade. Which side is so disadvantaged depends not on structural factors in the economy but entirely on initial prices and the dynamics these initial prices ignite. These odd features of the CCE have lead many economists to consider them unreasonable.

2.2 Experimental Design and Procedures

In order to assess the empirical content of Gale's strange predictions, we examined a series of discrete laboratory markets parameterized as above. In each of 8 sessions[11] between 12 and 20 subjects traded for approximately 3 hours. In each session half of the subjects were assigned Even preferences and endowments and half Odd, forming a replica of the economy described above.

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[9] In our discrete version of the economy, once prices reach 2,800 the economy is in fact in a stable corner equilibrium.

[10] At a price of zero (infinity) there is in fact a continuum of allocations at which Even (Odd) gives away an amount of y (x) that satisfies Odd's (Even's) demand without reducing Even's (Odd's) endowment utility.

[11] There was also a pilot session which used different utility parameters.

		Primary Phase		Reversal Phase	
Session	Market Size	Price Control	Free Prices	Price Control	Free Prices
1	12		1-14		
2	20		1-9	C: 10-12	13
3	20	C: 1-4	5-8	F: 9-14	
4	20	C: 1-11	12	F: 13-14	
5	20	F: 1-8	9	C: 10-14	
6	20	F: 1-7	8-9	C: 10-14	15
7	20		1-5	C: 6-14	15
8	16		1-4	C: 5-13	

Table 1: Summary of session design.

Sessions were divided into a sequence of 13-15 trading periods each lasting 6-15 minutes. Period lengths within each session were gradually decreased as subjects became more comfortable with the trading environment.[12] Trade was conducted via computerized continuous double auction using MarketScape software. Traders bought and sold units of y using the numeraire x as the medium of exchange. Subjects tracked their potential earnings using a special graphical program implemented in Excel that allowed them to visualize their induced indifference curves and activity rays and quickly calculate the payoff consequences of prospective trades.

As with most market experiments, implementation was via stationary repetition. At the end of each period subjects earned cash payments (paid at the end of the session) based on ending allocations and their induced utility functions, and allocations were returned to endowment levels for the next period of trade. Stationary repetition is especially useful in our study because tatonnement dynamics assume price adjustments at a fixed endowment. By resetting endowments at the beginning of each period, we can neatly map Walrasian predictions onto the sequence of period average prices.

The session design is summarized in Table 1. Each session opened with a block of periods that we call the Primary phase. In all but one session, we later attempted to reverse dynamics from the Primary phase, switching the sign of excess demand using

[12] Specifically, there was a paid training period of 15 minutes. Then the first "real" period was 15 minutes long, the second 12 minutes, the third 10 minutes, the fourth 8 minutes, and the rest 6 minutes.

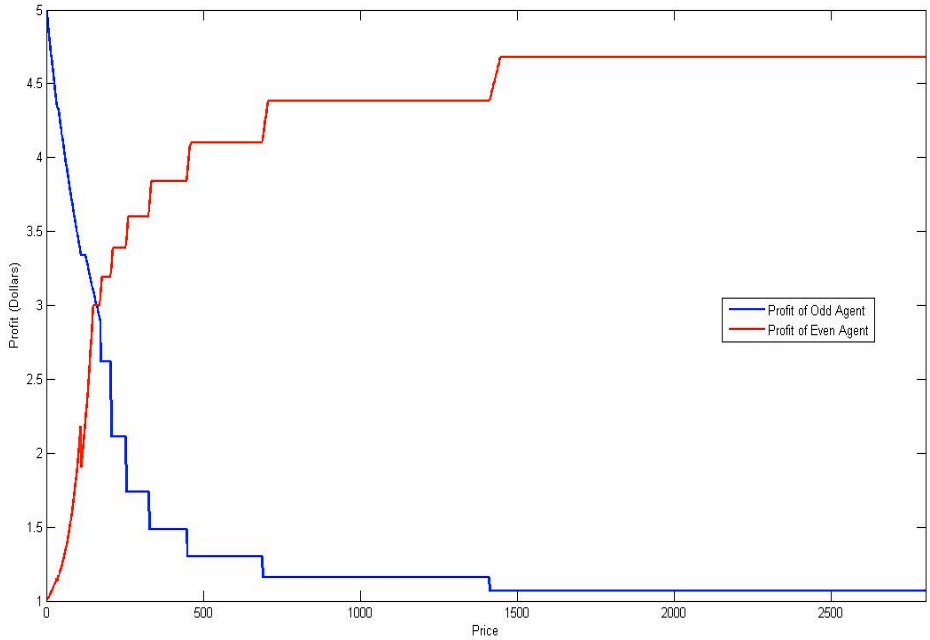

Figure 2: Total predicted earnings of both sides of the market as a function of price.

price controls. We refer to these later periods as the Reversal phase. In half of the sessions we allowed Primary phase prices to initiate freely. We call these Free sessions. In the other half we controlled the sign of initial excess demand using price controls; we call these Control sessions. In several sessions, time permitting, we lifted price controls in the final period of the Reversal phase.

Subjects received extensive training and instruction prior to trading. Instructions concerning both the character of preferences and the details of the MarketScape interface were read aloud to subjects. After reading instructions subjects engaged in a period of paid trade at a fixed price, allowing subjects to learn how to calculate earnings and submit orders without being allowed to engage in the strategy of setting prices. This gave them experience with their induced preferences and with the mechanics of the double auction.[13]

[13] We varied the training period price from session to session; 120 in session 1, 90 in sessions 2-4, and 275 in sessions 5-8. Thus half of the training prices were initiated in a region of negative excess demand, the other half in a region of positive excess demand. While the training price may influence the initial prices in "real" period 1, our objective is to study the evolution of prices over time, wherever they happen to start; disequilibrium theory is generally silent on where initial prices come from (in fact, training prices turned out to be a fairly weak predictor of subsequent "real" period 1 prices, as two sessions almost immediately "jumped" out of the signed region of excess demand in which they were trained). Training periods are standard practice in general equilibrium experiments (e.g., Anderson et al., 2004).

The preferences we induced (described in the previous section) are modifications of the ones described in Gale (1963). These modifications were made to achieve several goals without violating the salient features of Gale's economy. First, in order to keep orders discrete while maintaining a fine price grid, the numeraire good x is scaled to a much larger number of units than the commodity good y -- a conventional normalization of the Edgeworth box in general equilibrium experiments. We also included an additive constant in the utility function (not in Gale's original economy) that allows us to keep y orders reasonably low while maintaining a relatively narrow ICE price tunnel. Economies with lower trading volumes take less time to clear, allowing us to run shorter (and thus more) periods.

We also wanted to set an exchange rate that equalized total payoffs at all three equilibria and created symmetry in relative inequalities at the two corner equilibria. To accomplish this we induced a non-linear strictly monotone increasing util-to-dollar exchange rate that resulted in the map from price to profit (under the assumption of myopic optimal trade) shown in Figure 2. At the ICE price of 158 (and corresponding ICE allocation), subjects of both types earn \$3 per period. At her "good" corner equilibrium a subject would earn \$5, while a subject would earn \$1 at her endowment and in her "bad" equilibrium. In the figure can be observed a rough symmetry between the two subject types with respect to the marginal impact of price changes on myopic optimal profits; any price change is an approximately zero sum transfer from one subject type to the other, so we do not induce a bias for one subject type to care more about certain price changes than the other type.

Subjects do not see the induced utility or exchange rate functions, they only observe the dollar payments associated with benchmark indifference curves and have a calculator that converts any consumption bundle to dollars (and that plots its associated indifference curve).

2.3 Experimental Questions

Our design permits investigation of four main questions. The first is whether the interior competitive equilibrium range is behaviorally stable. When prices are in the ICE, do they stay there? Do prices outside of the ICE move towards the ICE over time?

Question 1: Is the interior CE behaviorally stable?

If the ICE is unstable in the sense that prices are not drawn to it, it is still possible that prices could show signs of global instability by failing to follow a clear trajectory. Our second question is therefore whether there is a strong (and direct) correspondence between the sign of excess demand and the direction of period to period price adjustments as tatonnement predicts:

Question 2: Are price dynamics Walrasian?

Even if price dynamics are roughly Walrasian, it seems unlikely that prices will continue moving to the extreme corner states predicted by the theory. It is the prediction of corner convergence that seems most implausible ex ante and our third question is whether this implausible prediction bears out in the data.

Question 3: Do prices reach corner equilibria and are these equilibria behaviorally stable?

We answer these three questions using the Primary phase data. Our final question is whether we can actually reverse the trajectory of prices by exogenously changing excess demand. An alternative hypothesis is that Primary dynamics, once established, are sticky.

Question 4: Can Primary phase price dynamics be reversed?

3. Results

3.1 Primary dynamics

Figures 3 and 4 display transaction prices from each session of the experiment. Periods are separated by vertical gray lines, phases by vertical black lines, and the ICE bounds are demarcated by horizontal dashed lines. Red dashed lines represent price controls. We focus on the Primary phase dynamics and later consider the Reversal phase.

In session 1 prices initiate within the ICE bounds and stay centered there for the remainder of the session. In every other session prices begin outside the ICE and never enter its bounds (one early transaction in session 2 notwithstanding). Indeed, as we'll confirm below, Primary prices always trend away from the ICE from period to period: When prices initiate above the ICE, they explode to an order of magnitude above it. When they initiate below, prices collapse to nearly (and sometimes exactly) zero. Prices within period also show a systematic tendency away from the ICE. These tendencies match Walrasian predictions; prices above the ICE generate positive excess demand and prices below negative excess demand. These observations answer our first question and provide us with a first result:

Result 1: *The interior competitive equilibrium is behaviorally unstable.*

Free session excess demand is endogenous, making it difficult to be sure it is causally related to price movements. Prices that initiate at positive excess demand also initiate high; perhaps the climbing prices we observe in Free sessions reflect a dynamic tendency that codetermines initial price and the price gradient. It is possible that excess demand is not in fact causally related to excess demand. [14]

To better identify the relationship between excess demand and price, we exogenously controlled and varied initial excess demand in half of our sessions (session 3, 4, 5 and 6).[15] In sessions 5 and 6 we used price floors in the first few periods to force

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[15] Al-Ubaydli et al. (2009) use a clever (and different) experimental intervention to rigorously circumvent endogeneity problems in identifying price dynamics.

excess demand at initial prices to be positive. In sessions 3 and 4 we forced prices to initiate with negative excess demand using price ceilings. Dynamics in these sessions are also Walrasian; sessions with price ceilings below the CE have prices dropping towards 0 while sessions with price floors above the CE have prices that rise far above the ICE bounds. Moreover, in each of these sessions we lifted price controls and prices continued both between and within period on their original trajectories, sometimes after a brief but unsuccessful surge in the opposite direction. Via exogenous treatment variation we are able to infer that dynamics are in fact caused by excess demand.

To test the Walrasian hypothesis more formally we calculate, for each session, the Mann-Kendall $\tau \in [-1,1]$, an ordinal, non-parametric measure of trend, for weighted average price across periods.[16] In sessions in which initial weighted average prices are at positive excess demand τ is nearly 1, indicating strong positive price trend, while in sessions with negative excess demand τ is close to -1. These measures are significantly different from zero at the 5 percent level in sessions 2-7. In session 8 τ is very close to 1 (0.999), but the small number of Primary periods allows us confidence at only the 10 percent level. These statistics generate our second result:

Result 2: *Primary disequilibrium price dynamics are Walrasian. From period to period, prices move significantly in the direction of the sign of excess demand.*

Within period, endowments adjust at each transaction price in the double auction so theoretical models of price dynamics within periods are difficult at best. For example, tatonnement models typically assume stationary endowments Nonetheless, in Figures 3 and 4 the trend in prices within period is obvious: Under positive excess demand, prices virtually always rise within period, and under negative excess demand they nearly always fall.

To check this more formally, we calculate the Mann-Kendall τ for prices and time within period, for each disequilibrium period in the Primary phase. In all Primary phase periods but one (period 9 of session 6) we observe significant trend within period.

[16] Similar results obtain using cardinal correlation measures such as Pearson's ρ.

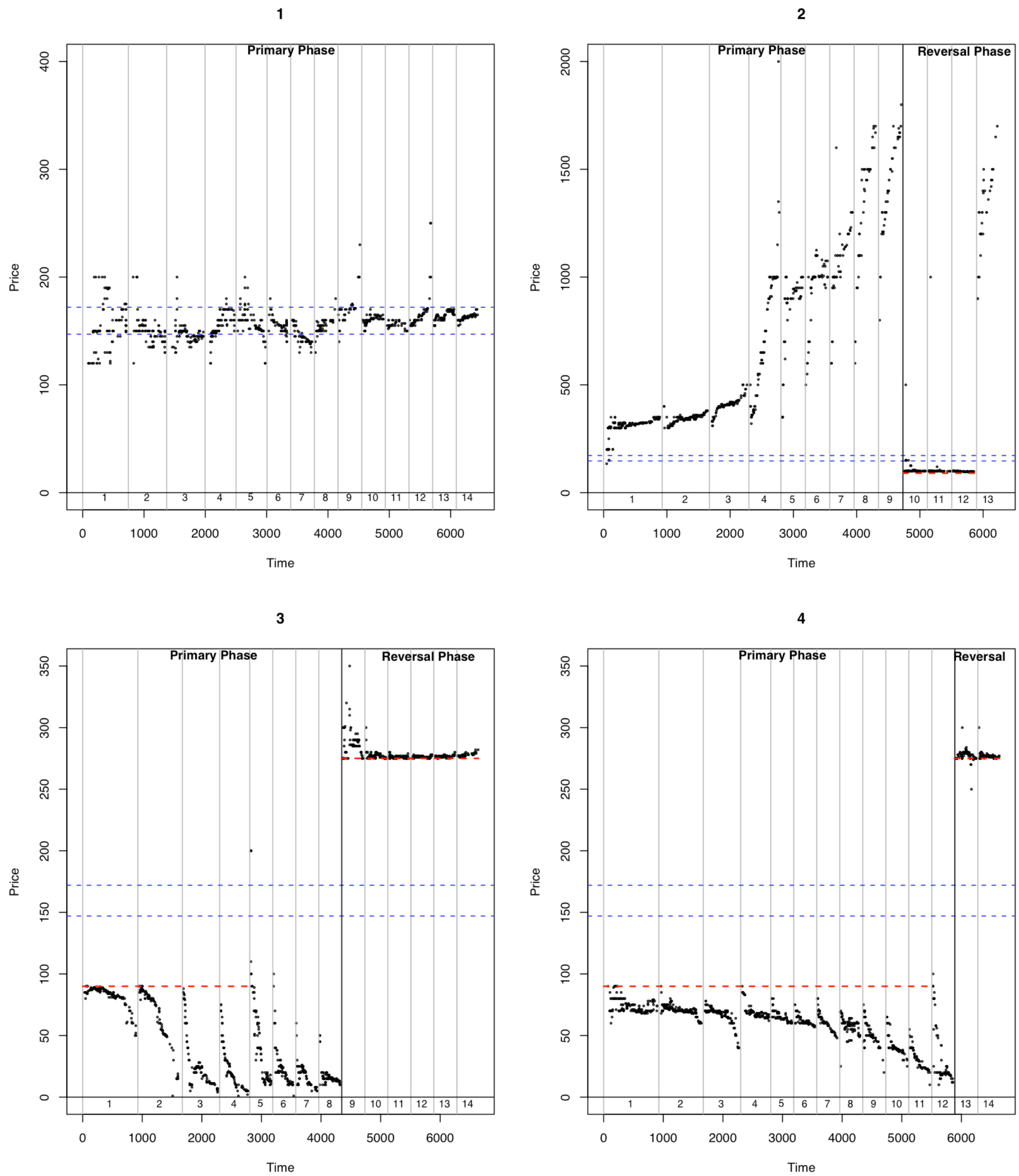

Figure 3: Price series from sessions 1-4.

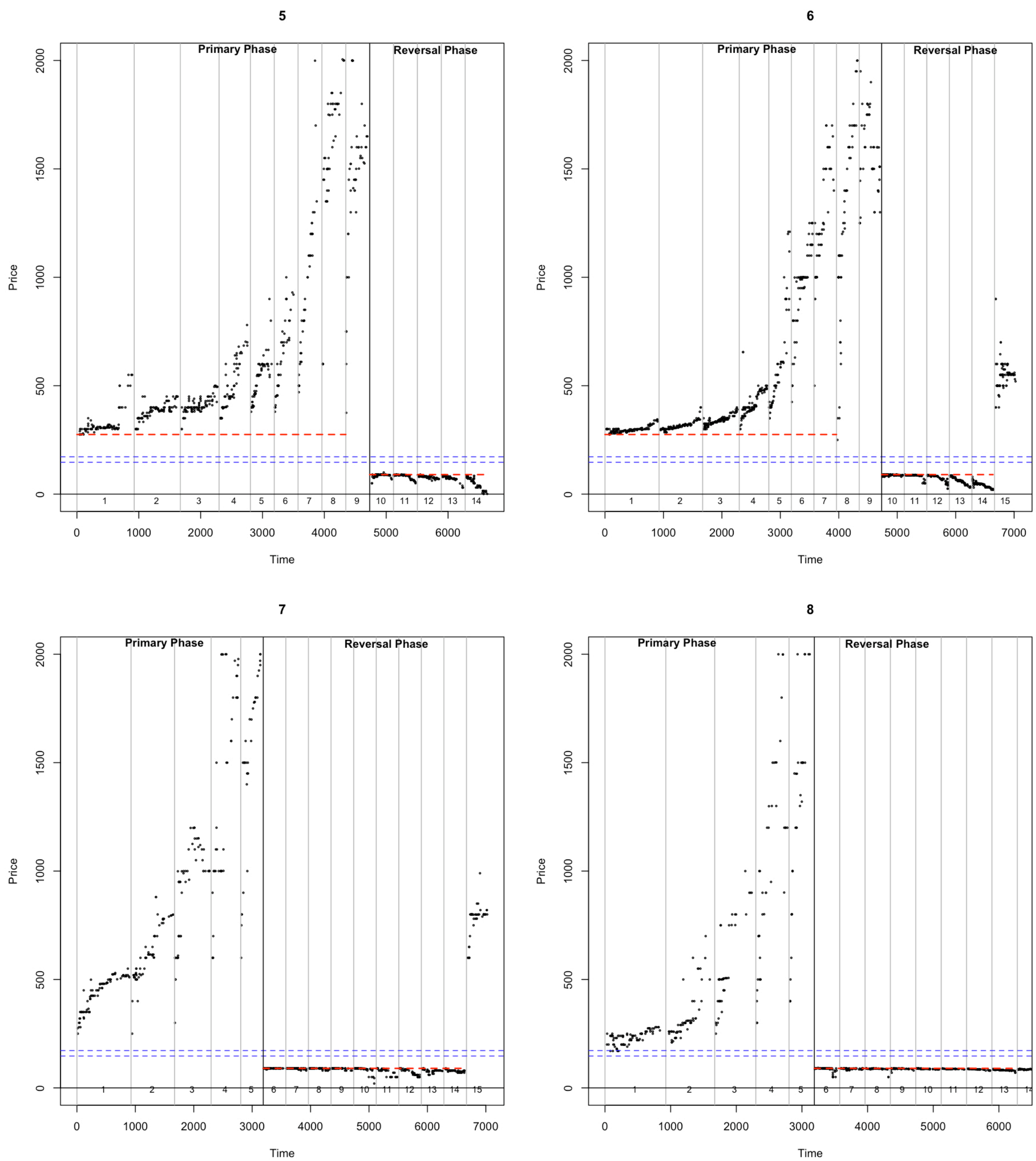

Figure 4: Prices from session 5-8.

In these periods with significant trend, prices universally match excess demand at both the current and previous average price. We document this as a further result:

Result 3: *The sign of disequilibrium price movements within period match the signs of excess demand.*

3.2 Corner Convergence

Disequilibrium prices show a strong tendency away from the interior equilibrium and towards the corner equilibria. How close to the corner equilibria do our markets come?

Our restriction of trade to the integer grid actually implies finite $p \geq 2800$ are competitive equilibrium prices, where each Odd (Even) subject buys (sells) one unit of y. In such equilibria the payoff to each Odd (Even) subject is \$1.07 (\$4.68); recall that subjects earn \$1 each at their endowments, so Odd is very nearly supplying good x for free. Thus prices of $p \geq 2800$ would certainly suggest "corner" prices.

But consider optimal symmetric (by type) trade at a price less than 2800 but still "large". For $p \geq 1446$ the profit of each Even subject is \$4.68, the same as in the (discrete) "corner" equilibrium (see Figure 2). Therefore there should be no pressure from the supply side for prices to go any higher. However, at $p=1446$ the Even subjects supply one unit of y each while the Odd subjects demand three units, so there is substantial excess demand.

How salient is this excess demand? When $p=1446$, if the demand of an Odd subject were fulfilled she would earn \$1.28, but under optimal symmetric trade (so that she is only able to acquire 1 unit of y) she earns \$1.07. Therefore the excess demand is worth \$0.21. Since symmetric optimal profits are constant for $p \geq 1446$ and excess demand is worth a fairly small amount (\$2-3 over the course of an entire session), $p=1446$ seems like a natural benchmark for convergence to a corner price (and 7 cents a benchmark corner trading profit for a price-disadvantaged subject). It is worth noting that the "demanded" profit for Odd subjects shrinks steadily as prices increase in the interval $p \in [1446,1643]$, above which Odd demands profit of \$1.16 for all $p \in [1643,2494]$ (so excess demand is only worth \$0.09 in this interval) before again declining towards \$1.07. Given this wide band (in prices) of profit non-monotonicity with such small "lost" profit attached to excess demand, $p \geq 1643$ is practically an equilibrium price.

By comparison, the payoff gradient on the price path to the $p=0$ equilibrium is relatively smooth. For symmetry we consider the near corner range to be any price that yields trading profit of 7 cents or less to the Even subjects, which corresponds to $p \leq 19$.

Figure 5 plots end-of-period prices (weighted average transaction price during the final minute of the period) from each session-phase combination.[17] For visibility across vastly different scales, the data is divided into a panel for periods initiating with positive excess demand and one with negative excess demand. Primary dynamics are plotted in blue; price control periods are linked by solid lines while free periods are linked by dotted lines. Horizontal black lines show the bounds for our "near corner" prices both on the high and low end. In all 7 disequilibrium sessions we observe prices entering this range by the end of the Primary phase. Mean prices therefore reach levels extremely close in payoff space to CCE prices.

[17] This gives us a view of combined within and between period dynamics.

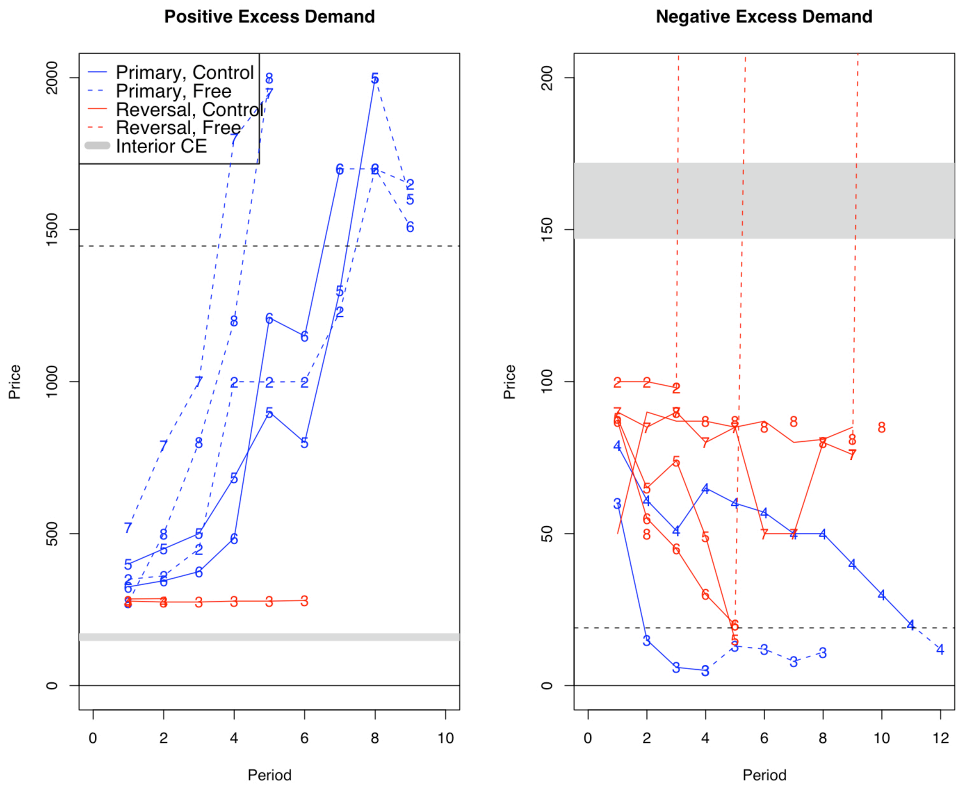

Figure 5: End of period prices by session.

Result 4: *Prices converge to close neighborhoods of corner competitive equilibrium prices.*

It is important to consider other measures of corner equilibration than price convergence. Optimal demands at convergent prices are useful, but unavoidably conceal heterogeneity inherent in the double auction institution that particularly matter in general equilibrium settings. Double Auction markets are cleared not at a single price within-period but at a host of prices evolving over the period's length. Moreover, subjects need not submit optimal demands at each transaction price. Indeed it is typical to observe individual subjects making multiple trades at multiple prices over the course of a single period.

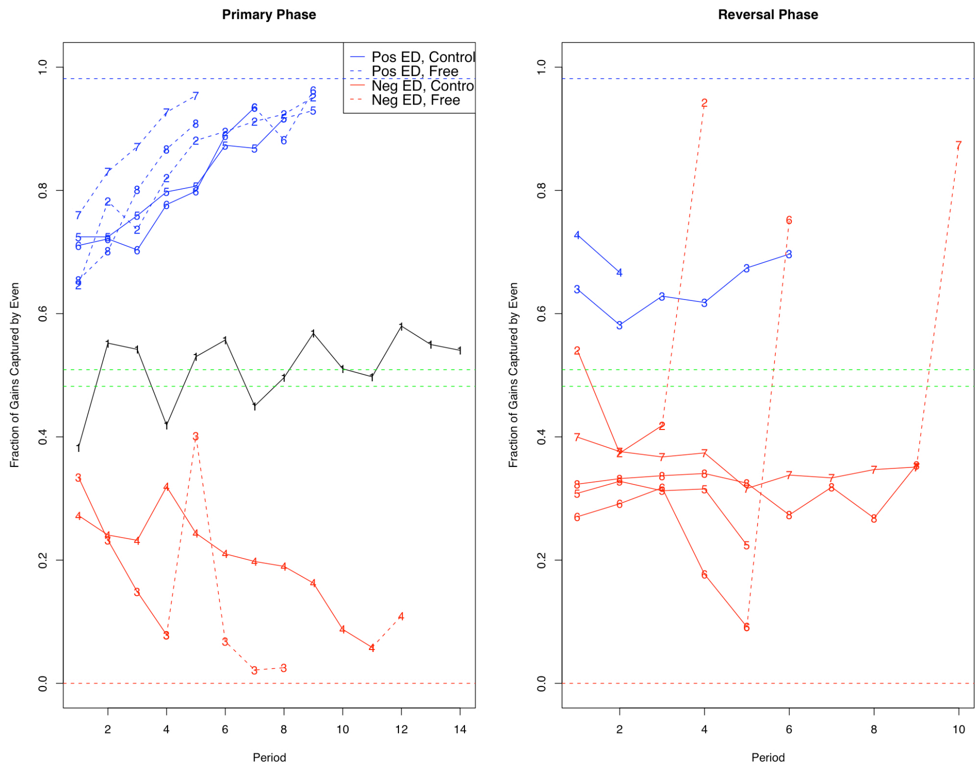

Figure 6: Even's average share of gains from trade by phase, period-within-phase and session.

We can sidestep these problems by directly studying the final allocations achieved by the market; how "close" are these allocations to corner equilibrium allocations? A simple metric of linear distance in the Edgeworth box is of limited value because of the highly nonlinear nature of rewards in this space, because there are a large set of equilibrium allocations from which to benchmark distance, and because allocations are restricted to a discrete grid, (meaning a range of prices typically support identical levels of demand and supply, and a single price tick can imply a discrete jump in excess demand). An appealing alternative is to examine inequity across player types in the achieved gains from trade (relative to the endowment), a one-dimensional measure that is increasing as allocations move from the ICE towards either CCE.[18] This measure is also

[18] Note that, as is clear from Figures 2 and 7, total gains from trade vary little at optimal or near optimal demands in our parameterization of the Gale economy; the main effect of a change in price on payoffs is in relative shares of the gains from trade earned by each side of the market.

a particularly strong test of corner convergence in that it measures the aspect of the corner equilibrium that seems least plausible, ex ante.

For each period, we calculate the share of total gains from trade achieved by the Even side of the market. At the upper corner (for $p \geq 1446$) the benchmark symmetric optimal measure is 0.981 while at the lower corner (for $p=19$) it is 0.019. In Figure 6 we plot these benchmarks as dashed horizontal lines. We also include the symmetric optimal benchmark gains at the upper and lower bounds of the ICE in dashed green.

For each session and period we plot the evolution of Even's average share of the market, with a separate panel for the Primary and Reversal phases. Primary phase results are compelling. In session 1 where prices initiated in the ICE tunnel, gains from trade are close to evenly split. In all disequilibrium periods, prices move decisively[19] away from equity, generally ending with one side of the market earning in excess of 95% of the gains from trade. These shares come extremely close to those obtaining at the corner equilibrium.[20]

[19] In session 3 there is a temporary spike towards equity when price controls are relaxed but shares quickly plummet back towards zero.

[20] Note that although earnings shares change dramatically over the course of sessions, as predicted total realized payoffs barely change at all. Figure 7 plots the median per-pair payoff by session, phase and period-within-phase. Green horizontal lines show the upper and lower bounds of predicted payoffs based on symmetric optimal orders. Per pair payoffs fluctuate little and show no systematic trend. Moreover they indicate markets are relatively efficient in that subjects typically extract nearly all of the gains from exchange.

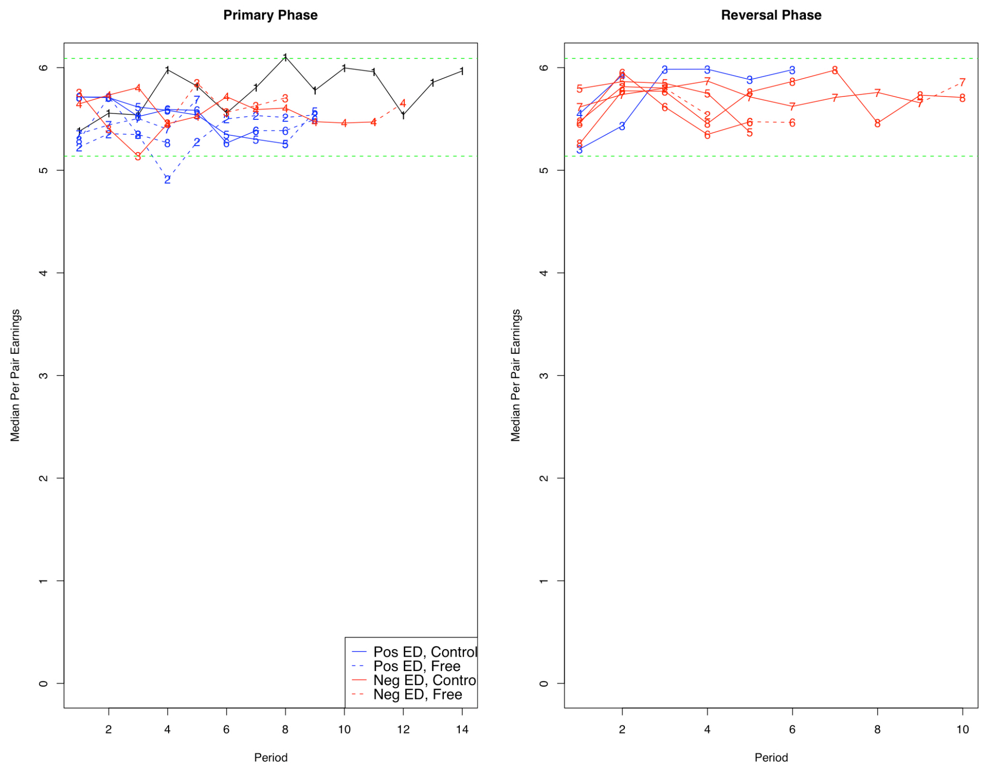

Figure 7: Median per pair dollar earnings by phase, period within phase and session

Result 5: *Out of equilibrium dynamics lead one side of the market to capture nearly all gains from trade, eventually more than 20 times that of the other side. These inequities are close to those predicted at corner equilibria.*

We ended the Primary phase at arbitrary times in most sessions. Had markets prices settled down by the time we ended the primary phase or were they still moving closer to the corner? The limit order book suggests that we typically closed the Primary phase before Walrasian dynamics had finished influencing the evolution of market prices. In virtually all sessions a large number of competitive orders were left in the limit order book from the price-disadvantaged side of the market, suggesting that the disadvantaged side of the market was prepared to offer more extreme prices than those observed by the phase's end. So while we demonstrated convergence to a reasonably tight neighborhood

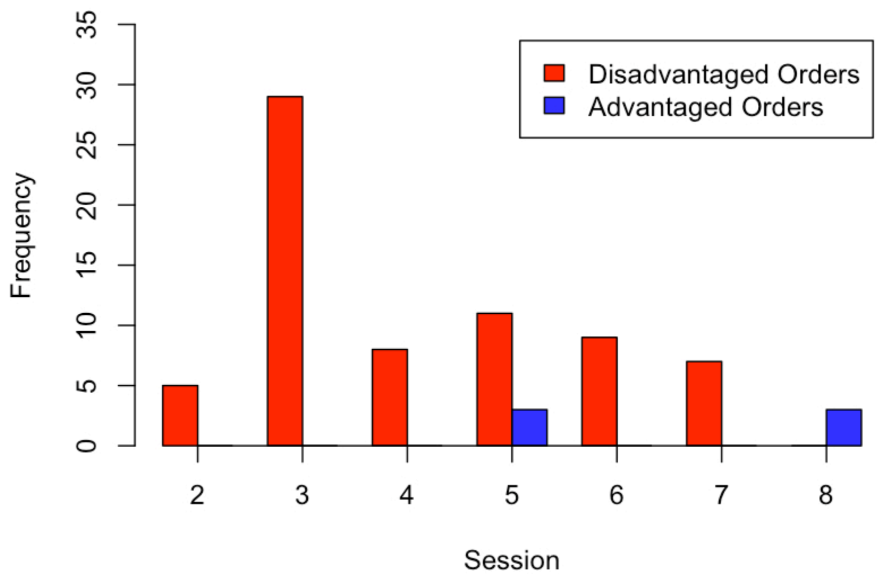

Figure 8: Competitive units in the limit order book at the end of the final period of the Primary Phase.

of corner equilibria in Result 4, there are signs that these economies may have moved even closer to the corner if we had increased the length of the Primary phase.

Figure 8 plots the number of "competitive" units left in the limit order book on the "advantaged" and "disadvantaged" sides of the market (i.e., an advantaged (disadvantaged) order is an ask (bid) in high price periods and a bid (ask) in low price periods). Here competitive is defined as a bid (ask) no more than 10% below (above) the final transaction price of the period.[21] In most sessions[22], the Primary phase ended with a large number of unrequited competitive orders submitted by the disadvantaged side of the market and very few orders left from the advantaged side. Such unfulfilled expressed

[21] Results are robust to choice of the definition of "competitive" At the 20% criterion a cluster of advantaged units (bids) in Session 4 becomes competitive and is substantially greater than the number disadvantaged units (asks). Otherwise disadvantaged orders dominated in all sessions for all levels of competitiveness, the small imbalance in session 8 notwithstanding.

[22] The only exception to this rule was session 8. In this session the order book in the final period of the Primary phase was dominated by disadvantaged units, as well (31 to 3), but because prices moved so fast near the end of the period (the weighted average price in the period was 1292 but the final transaction price was 3000), the bids left in the book at the end of the period were not categorized as competitive by our metric.

demand is a primary driver of price changes across periods; unaccepted bids at the end of one period are likely to lead to higher bids in the subsequent period. We therefore interpret this as suggestive evidence that trading prices likely would have continued to move further from the ICE had we extended the duration of the Primary phase with additional periods.

3.3 Persistence of Dynamics

Relatively quick near-corner convergence in most sessions inspired us to investigate the durability of dynamics. Once established, can dynamics be easily reversed? In order to find out, we attempted to reverse Primary dynamics by imposing price controls, forcing a reversal of excess demand in the 7 "corner" sessions. In sessions 2, 5, 6, 7 and 8 price ceilings at p=90 created excess supply in the Reversal phase while in sessions 3 and 4 price floors at p=275 generated excess demand.

Evidence that dynamics were reversed by the forced changes in excess demand is mixed. In sessions 5 and 6 prices drop substantially over time suggesting successful reversal of dynamics. In sessions 3, 7 and 8 we observe prices hugging the price constraints suggesting the opposite (the price-controlled Secondary Phase was only 2-3 periods long in session 2 and 4, too few periods to assess reversal). In fact, in sessions 7 and 8 the Reversal phase consisted of more periods than the Primary phase, and yet prices hugged the price ceiling throughout.

In three sessions (2, 6 and 7) we lifted the price controls of the Reversal phase in the final period. The effect is most clearly documented in the right hand panel of Figure 5, where red dotted lines show the price path after price controls are lifted. In each of these periods we observe prices leaping far past the ICE to the region of excess demand established in the Primary phase. Such explosive adjustments to price control removals have been reported elsewhere in the experimental literature. Moreover prices within these periods continue moving upwards within the period in accordance with the dynamics established in the Primary phase. Another view of the effect of lifting the price controls can be found in Figures 3 and 4.

Amazingly, this happens even in session 6 where we observe an initial apparent reversal of dynamics while price controls persist. After price controls are lifted this new trajectory is abandoned and the pattern from the Primary phase is reestablished.

Result 6: *Dynamics once established are typically sticky and difficult to reverse with price controls.*

The difficulty of reversing dynamics demonstrates that price dynamics do not follow mechanically from excess demand. Whatever channel ties excess demand to price motion, the relationship is not a behaviorally trivial outgrowth of the strategic environment. There is some evidence (documented in the Appendix) suggesting that members of the advantaged side of the market overtrade in the Reversal phase to a much greater degree than in the Primary phase. This overtrading seems to be a proximate cause of the failure of prices to adjust in the reversal phase.[23] Our data, however, is too heterogeneous to form the basis for a credible theory of dynamics reversal. We leave the important work of systematically studying dynamics reversals to future research.

4. Discussion

Experiments are often appealing because they can surprise us. Frequently they do this by providing us with evidence that intuitive theories fail to explain behavior (i.e. expected utility). But they can also surprise us by showing us that highly counterintuitive theories succeed in predicting behavior. We have taken one of the more counterintuitive theoretical examples in economics and provided evidence that it precisely anticipates outcomes in laboratory markets. In the process we have provided perhaps the strongest evidence to date that Walrasian-like dynamics govern price adjustment in general equilibrium environments.

[23] Overtrading may be a special consequence of Leontief economies. With Leontief preferences, payoffs are relatively flat within much of the Pareto set, so traders have relatively low incentives to avoid overtrading. Thus overtrading can be rationalized as a relatively cheap option on within-period price reversals, which seems plausible given that it becomes more prevalent after subjects have become experienced in the mechanics of trading in a double auction institution.

Prices in our markets generally initiate outside of the economy's conventional and equitable competitive equilibrium. When they do, as predicted, prices explode to more than ten times the centroid of the ICE or collapse to nearly zero, driven by nothing else but dynamic forces set off by initial prices. These forces continue to exert influence even at extreme prices, resulting eventually in one side of the market giving away its endowment for pennies and the other harvesting nearly all of the gains from exchange.

Although the extremity of the results seemed unlikely ex ante (at least to two of the authors), they come about for ultimately economically sensible reasons. Indeed the Walrasian dynamics driving our results are simply a general equilibrium analogue of our standard textbook explanation of equilibration in simple supply and demand markets. Because of the Gale economy's powerful income effects, prices greater than the interior CE induce buyers to demand more units than sellers are willing to supply. Buyers, scrambling to acquire scarce units, bid prices up, and because no fixed point exists to extinguish this process prices continue moving ever upward.[24] Symmetric arguments hold when prices begin below the interior equilibrium; low prices cause a glut of supply and drive prices downwards in a process that only ends at prices near zero.

We agree with Chipman's (1965) assessment that the Gale example "is a special limiting case, and should not be taken too seriously as providing an illustration of any real situation." Our interest in the economy derives not from its realism but instead from the peculiar stress it imposes on Walrasian predictions. If dynamics consistently operate where outcomes are so extreme, it seems, they will operate anywhere. It is partly because nature probably rarely produces environments this tough that a laboratory test is important.

The implication of our research is that classical classifications of the stability of equilibria are empirically meaningful, even when markets are not cleared in a tatonnement institution and even when the implications of stability classifications are highly counterintuitive. We believe our paper, in conjunction with the handful of recent papers on the topic, represents a broad empirical vindication of much classical general equilibrium theorizing on out-of-equilibrium dynamics.

[24] Of course in our discrete implementation the incentives to continue driving prices eventually go to zero.

Much work remains to be done in this area. A secondary result from our project (and Plott, 2000) is that dynamics do not readily or reliably adapt to shocks to excess demand. Can all subjects be convinced of the integrity of the price control, or is the memory of previous extreme prices too much for some subjects to ignore? Are there other causes of price stickiness at a control that is not binding under tatonnement?

Our experiment focuses on qualitative aspects of dynamics. As it turns out the special character of excess demand functions in the Gale example – excess demand increases as the economy approaches equilibrium – makes it difficult to estimate the quantitative relationship between excess demand and price movements. High excess demand should induce faster price movement, but here high excess demand also coincides with diminishing marginal utility of consumption, leading to inevitable slowdowns as prices approach corner equilibria. Studying less extreme, smooth variations on the Gale economy (e.g., CES preferences or Marshall's (1879) example) would avoid this problem and enable quantitative characterization.

Finally, our experiment is designed to show off the character and test the limits of dynamics. Our units of observations are market cohorts; strong interdependence between subject decisions and endogeneity of key variables limit us from saying much about individual decision making with much confidence. An important unexplored frontier for research is to use novel designs that exogenize excess demand (perhaps via carefully controlled shocks to endowments or preferences) within period and thereby enable credible characterizations of how individual decision making operates in these markets.

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Appendix: Sticky Prices in the Reversal Phase

For a given period, consider the optimal excess demand for each subject at the final allocation under the period's weighted average price. Note that an Even (Odd) subject only has positive (negative) excess demand if he trades across his activity ray. Ignoring for the moment the possibility that a subject might trade across his activity ray because he anticipates a within-period change in the direction of prices (e.g., buy low – sell high), then all period-ending excess demand (supply) in high (low) price periods should come from Odd (Even) subjects, and there should be no excess supply (demand). That is, there should only be individual excess demand or supply (not both), and it should come from price-disadvantaged subjects.

As it turns out, under-trading by price-advantaged subjects and over-trading by price-disadvantaged subjects is relatively small, so non-zero individual excess demands at the end of a period are indeed generally of the same sign (positive in high-price periods, negative in low-price periods). However, there is typically a substantial volume of individual excess demand that comes from price-advantaged subjects. As mentioned above, this "over-trading" can be rationalized as expecting a change in the trajectory of prices within period. For example, an Even subject might see rapidly rising prices as a bubble, so he oversells good *y* in anticipation that he will be able to buy units back cheaply later in the period. With Leontief preferences, over-trading is a relatively cheap option on within-period price changes.[25] Further, the over-trading is mostly committed by just 1-2 subjects. The median absolute value of individual excess demand of price-advantaged subjects is less than or equal to one in 97% of all periods, so the median price-advantaged subject nearly always trades to his activity ray and stops.

[25] The more "extreme" prices become, the smaller the profit penalty an advantaged subject pays for crossing his activity ray. For example, suppose the price is 20 (a very low price). A myopic optimizing Odd subject will obtain the bundle (5400,15) and earn a profit of $4.60 for the period. An Odd subject who "over-buys" by 15 units of Y (a lot!), so that he ends up with (5100,30), will still earn $4.06. What is the potential reward? If prices move to the high range, say 1500, the subject could earn $111.36 for the period by selling back 5 units of Y!

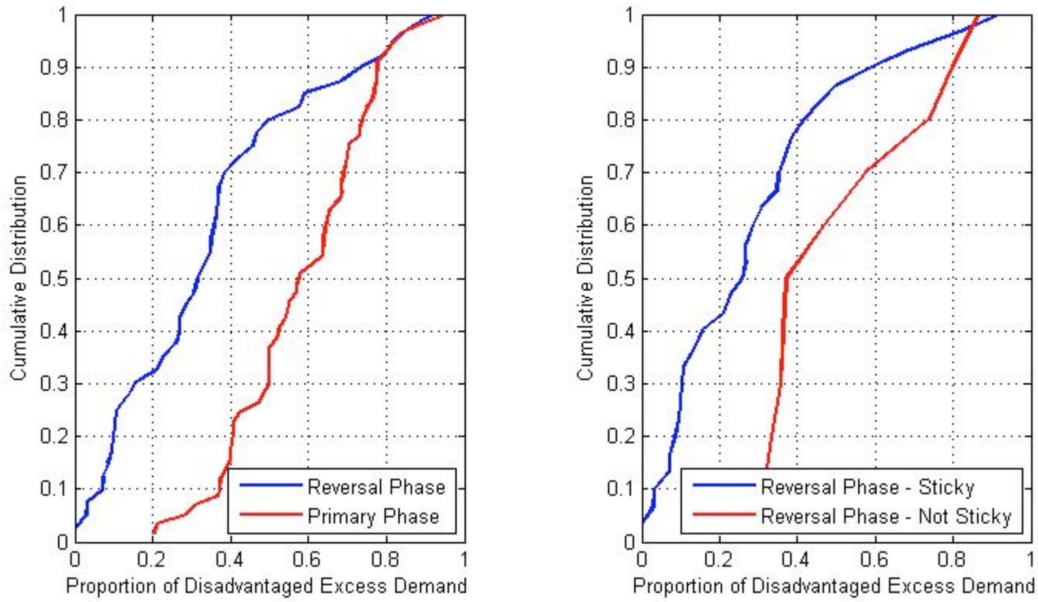

Figure 1A: Proportion of excess demand generated by price-disadvantaged subjects.

Over-trading by price-advantaged subjects may potentially explain the price-stickiness we observe in the Reversal phase. For each period of sessions 2-8, we partition subjects by type, and sum the positive (negative) individual excess demands in high (low) price periods. We then calculate the proportion of these excess demands that come from price-disadvantaged subjects. What we find is that this proportion is substantially larger in the Primary phase than in the Reversal phase, and it is substantially larger in the two Reversal phase sessions where we observe prices moving towards the corner equilibrium (sessions 5 and 6). In Figure 1A we've pooled the proportion individual excess demand that comes from price disadvantaged subjects across sessions. From the left panel of the figure we observe that in the median period of the Primary pool, nearly 60% of excess demand is generated by price-disadvantaged subjects, double the amount generated by price-disadvantaged subjects in the median Reversal period. The distributions remain far apart throughout; clearly, over-trading is much more prevalent in the Reversal phase, despite the fact that subjects in this phase are much more experienced.

In the right panel we consider only the Reversal phase periods in which the price control was in place, and put sessions 5 and 6 (where there was strong evidence of price reversals) in one pool and the other sessions in a separate pool. We observe a similar pattern to the left panel: Over-trading was more prevalent in the sticky sessions than in non-sticky sessions. Over-trading did tend to be greater in the non-sticky Reversal phase sessions than in the Primary phase, but this fact is intuitive considering that price dynamics were slower than in the Primary phase.

Why should the excess demand of over-trading subjects not drive prices in the direction of tatonnement? First consider a high-price period, and an Odd (disadvantaged) subject who has positive individual excess demand. We may reasonably infer he would like to have purchased more of good y but was unable to do so. It is his failure to fill his full order that invokes the spirit of tatonnement; he is likely to be more aggressive in his activity the following period in order to (hopefully) fill his full order, helping to push prices further upward.

Now consider an Even subject with positive individual excess demand in a high-price period. If his "over-trading" was intentional, we can infer that he weighted the probability of prices dropping by the end of the period sufficiently high to risk over-selling. If he employs the same strategy in the following period, his "excess demand" is by definition but not intention. He will not exhibit increased aggressiveness in his buying behavior like the aforementioned Odd subject (because he will not buy at all, only sell), and he will not reduce his (over) selling behavior by assumption, so his individual excess demand at the end of the period will have no impact on subsequent prices. Only if he views his over-trading as a mistake will his behavior in subsequent periods contribute to tatonnement price pressure.

Given their experience with extreme prices in the Primary phase, it is natural that subjects would consider the possibility of returning to this price region during the Reversal phase. But it was impossible with the price control. At the beginning of each low-price Reversal phase period, the experimenter announced, "All orders must be posted at a price less than or equal to 90. This restriction will remain in place for the entire period." (The high-price announcement was the same, but with orders restricted to a price greater than or equal to 275.) If these instructions were interpreted as credible, over-trading at the price control (where sticky Reversal prices remained) makes no sense at all; the option value for over-trading only kicks in if prices can break the control. Nevertheless in these phases one or two subjects always over-traded by a large quantity. One such subject (an Odd subject in session 7 who did not over-trade in the Primary phase) averaged an excess supply of 23 units during the Reversal phase! This subject was asked after the experiment if he realized he was over-buying. His reply (paraphrasing): "Absolutely. I thought prices would jump back up to where they had been at the beginning of the experiment and I'd make a lot of money." This subject either ignored the announcement that the price control would remain in effect until the end of the period or did not find it sufficiently credible to forego the relatively cheap option on a price reversal. It appears the idea that prices might go back to where they had been during the Primary phase of the experiment exerted substantial influence during the Reversal phase, even though they couldn't.