Open vs. Closed Rules in Budget Legislation:  
A Result and an Application*

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Abstract
Baron and Ferejohn’s (1989) divide-the-dollar game is one of the most cited bargaining models in political science. A main result of the model is that legislatures will prefer closed rules to open rules in distributive bargaining because closed rules eliminate the possibility of delay. However, open rules are far more prevalent in the budget process at both the federal and state level. This puzzle is resolved by considering a variant on Baron and Ferejohn’s model that permits spending to vary rather than being prefixed. Under these conditions, an open rule is almost always preferred to a closed rule by the legislature. This model is then applied to the problem of enforcing spending limits. I demonstrate that the legislature’s preference for open rules is so strong that it is typically unwilling to trade off a closed rule for lower levels of spending. This result has implications for the design and enforcement of rules in legislatures.

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1 Introduction

The rules determining voting on the floor of legislatures often influence policy outcomes. When a bill reaches the floor of the U.S. House of Representatives, battles have typically been waged during the creation of the rule governing debate on the legislation. A rule determines what amendments (if any) are in order, the order in which they shall be considered, the time for debate, and so on. While there are endless ways to structure consideration of a bill, it is useful to apply a simple distinction between open and closed rules. Closed rules permit a straight up-or-down vote on a piece of legislation, whereas open rules allow for amendments to the legislation.\(^1\)

If only one policy area is under consideration and legislators’ preferences are well-behaved, then an open rule is optimal from the perspective of the median voter, since his ideal point is always obtained if every legislator is allowed to make an amendment at some point. A closed rule, on the other hand, may permit a proposer (whether a committee or some other agenda setter) to prevent policy from being set at the median’s ideal point, even in infinite-horizon bargaining (Romer and Rosenthal 1978; Primo 2002).\(^2\) Despite the apparent advantage of open rules in legislative settings, restrictive rules have become more common in recent years (Oleszek 2001; Sinclair 2000). There are many explanations for this phenomenon. For instance, Gilligan and Krehbiel (1987) posit that restrictive or closed rules, which ostensibly give committees greater power, can be used to induce the committee to gather information and reduce the uncertainty inherent in policy-making: “[A]cting in its self-interest, the parent body often restricts its ability to amend committee proposals” (Gilligan and Krehbiel 1987, 288).

In contradiction to these findings, appropriations bills are typically considered on the

\(^1\) In the U.S. House, for instance, rules can be separated into open, closed, modified open, and modified closed (or structured).

\(^2\) Cho and Duggan (2003) demonstrate that even under closed rules, the median voter outcome obtains under some very weak conditions, provided that all legislators have a positive probability of making a proposal. Of course, in some cases, proposal power is prefixed, as in the case of Presidential appointments; it is in these cases that a proposer can shift policy away from the median.
House floor under open rules (albeit with some limitations), and open rules dominate in state appropriations (Grooters and Eckl 1998). An open rule has the potential to unravel carefully built coalitions on sensitive spending matters, so its frequent usage may seem puzzling at first glance. This paper suggests one reason for the prevalence of open rules—that it prevents the committee from obtaining unreasonable power over appropriations.

The budget process can be studied by focusing on its distributive aspects. Distributive goods benefit a particular geographic region but are funded out of general revenues, and they are sometimes derisively referred to as “pork-barrel” projects. Legislators in such a framework seek to maximize the value of the distributive good in their districts and minimize spending in other districts. In addition, legislators prefer projects that are larger than efficient, since each district pays only a portion of its project’s cost. An analogy is the tendency for restaurant patrons to select more expensive meals when the tab is split evenly than when separate checks are issued.

An example will help to fix ideas. A one-hundred-member legislature splits the costs for district-specific projects equally across districts. Suppose that the district benefit associated with a new government building that is \( x \) square feet is \( x \), and the total cost is \( 180,000x^2 \). The efficient building size is then 40,000 square feet. But since the district pays only one percent of the building costs, the legislator’s ideal project is a building that is 4,000,000 square feet, one hundred times larger than optimal, and the difference between a visitor’s center and a convention center.

One of the most-cited distributive politics bargaining papers shows that closed rules will be preferred by the chamber to open rules when a dollar is divided (Baron and Ferejohn 1989). In the Baron and Ferejohn model, a randomly-selected agenda setter proposes the division of a dollar. Bargaining continues, and a new agenda setter is chosen, until a proposal is voted upon favorably by a majority. Delay reduces the size of the dollar. Under a closed rule, agreement is reached immediately, while under an open rule, delay is possible. Because the full (discounted) dollar is allocated in the model, delay is the only feature of the model
that affects the game’s expected value. The closed rule’s sole advantage is in eliminating the possibility of delay.

This paper builds on the distributive politics literature and considers a model in which the budget is not prefixed. With this setup, open rules are preferred to closed rules, consistent with the empirical evidence. This result is substantively important in its own right, but it also allows us to consider an additional issue: the endogenous enforcement of budget rules. Specifically, inefficient projects often arise in distributive politics bargaining, and the model presented in this paper is no exception. The question is whether the legislature can mitigate the problem by trying to constrain spending through the use of spending limits tied to amendment rules. I show that this is possible in only limited circumstances, and that the endogenous enforcement of spending constraints will have to arise from some other mechanism. While endogenous enforcement is not possible in this case, this application serves as an example of an effective approach to the study of budgetary institutions: one which harnesses the existing structure of a legislature (which is assumed to be the equilibrium to some larger repeated game) to sustain endogenous enforcement of budget rules.

The study of endogenous enforcement requires assumptions about what is exogenous, or fixed, in a given institution. One perspective is that “all rule enforcement is endogenous, and that, just as the maintenance of informal agreement is always problematic, so is the enforcement of formal rules” (Calvert 1995, 78). The polar opposite perspective is what Calvert terms the “institutions as constraints” perspective, which views institutions as fixed rules that are perfectly enforced. While appropriate for studying the influence of rules on behavior, this approach is less effective for studying institutional choice.

There is a middle ground between evanescent institutions and fixed institutions. Diermeier and Krehbiel (2003) distinguish between institutional theories and theories of institutions. The former examines how a given institution influences behavior. The latter examines the selection and choice of institutions and their stability. However, Diermeier and Krehbiel suggest that it is sufficient for specific institutions to be objects of choice, so long as the
fixed institutions are understood to be constraints (2003, 132).

If the super-structure of a legislature (the committee system, floor procedures, the structure of amendment rules, etc.) is taken to be exogenous, then it can be utilized as a tool for enforcement of budget rules. A justification for this approach is that institutions are essentially repeated games in which defection is never the preferred strategy. Endogenous enforcement of budget rules occurs if the existing (equilibrium) institutional framework is successfully used for enforcement.

Of course, there are other ways, given a fixed institutional structure, that budget rules might be sustained. One approach to the study of institutions is to assume that external enforcement exists (e.g., Primo 2003). Assuming external enforcement is useful because it permits institutional analysis of either the impact of the rules or the principal-agent problems inherent in such enforcement.

Another approach is to study repeated games in a legislative setting, which allows for norms to be enforced endogenously. But rarely will a particular norm not be sustainable in some equilibrium, due to the folk theorem. For example, the norm of universalism, first suggested by Weingast (1979) and generalized by Shepsle and Weingast (1981), suggests that legislators in distributive politics settings will agree to form unanimous coalitions rather than minimum-winning-coalitions because of the uncertainty inherent in coalitions that do not include all members. It is possible in this case for inefficient projects to be doled out. Universalism, therefore, is a norm that explains inefficiency in distributive politics.

While Shepsle and Weingast (1981) did not use the argument of repeated games to argue for the norm, ultimately this is necessary for the norm to persist. The problem, however, is that this norm—call it inefficient universalism—is Pareto-dominated by other norms, such as efficient universalism, which can also be sustained in a repeated game. Efficient universalism results if all players receive the efficient project for their districts in every period. The puzzle,

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3Legislative bargaining naturally admits an overlapping generations framework, since legislative membership is not stable (e.g., Diermeier 1995), but folk theorem-type results hold for models with finitely-lived actors in an infinitely-lived institution as well (Kandori 1992).
then, is why inefficient universalism would persist when the Pareto-optimal outcome can also be sustained as an equilibrium norm.

The notion of norms and institutions as equilibria to repeated games is most useful, I argue, for justifying the exogeneity of certain structural features of a decision-making body, which in turn may aid in the endogenous enforcement of budget rules. In this paper I take one step in this direction and attempt to understand whether budget limits can be enforced by taking advantage of existing legislative organization. Specifically, I assume that amendment rules, once selected, will not be undone on the floor. The selection of amendment rules is then used as an enforcement mechanism for a budget rule. Though the mechanism is successful only under limited circumstances, this analytical technique is a productive way to proceed in thinking about the enforcement of budget rules.

This paper proceeds as follows. In section two, I describe the model. Section three addresses the influence of amendment rules on the distribution and size of projects. Section four introduces the problem of enforcement. Section five presents the results of the application, and section six concludes. Technical results and proofs are presented in the appendix.

2 The Model

2.1 Actors and Preferences

A legislature $L$ is comprised of $n$ identical legislators, where $n$ is odd for convenience, who have preferences over a vector of projects $X = \{x_1, x_2, x_3, ..., x_n\}$, $x_i \in [0, \infty)$.\(^4\) The legislator in district $i$ evaluates the benefits and costs of projects with the net benefit function

$$bx_i - \frac{1}{2}cx_i^2.$$  

A social net benefit function for the legislature is

$$\sum_{i=1}^{n} bx_i - \sum_{i=1}^{n} \frac{1}{2}cx_i^2.$$  

\(^4\)The closed rule, simple majority version of this model was first presented in Baron (1993).
The net benefit functions imply that the efficient project scale is $\frac{b}{c}$ for each district and that the net benefits of a project decline monotonically as projects move away from the efficient scale. Put another way, marginal utility is declining in project size (as evidenced by a negative second derivative of the net benefit function). For example, suppose that $b = c = 1$. Then the optimal project size is 1, which provides a net benefit $1 - .5 = .5$. Note that the marginal utility from moving from a size 0 project to a size .5 project is $\frac{3}{8}$, but the marginal utility of moving from a size .5 project to a size 1 project is only $\frac{1}{8}$.

Because the cost function $\frac{1}{2}cx_i^2$ is convex in project size and there are no fixed costs, a series of smaller projects that collectively total size $z$ will be less expensive combined than one large project of size $z$. In the case of a quadratic function, this is easily seen by noting that the square of a sum must be greater than the sum of squares. This fact, combined with a linear benefit function, suggests that a series of smaller projects produces larger social net benefits than one large project.

The convexity of costs is necessary to avoid degenerate preferences (i.e., either a preference for no project or an infinitely-large project).\(^5\) The convexity of the cost function, $\frac{1}{2}cx^2$, and the concavity of the net benefits function, $bx - \frac{1}{2}cx^2$, are central to the results.

The net benefits from $X$ for legislator $i$ are

$$NB_i(X, b, c) = bx_i - \frac{1}{2}c \sum_{j=1}^{n} x_j^2.$$  

The benefits of the projects do not spill over into other districts (i.e., no positive externalities are present), and districts share the total cost of projects equally. Given this function, legislator $i$’s ideal vector of programs consists of $x_i = \frac{nb}{c}$ and $x_j = 0 \forall j \neq i$. The legislators in this model, therefore, prefer projects that are larger than efficient (and therefore cost more than is efficient). Further, the efficient outcome is for all legislators to receive a project of size $\frac{b}{c}$.

\(^5\)A function $f(x)$ is strictly convex if $f(\lambda x_1 + (1 - \lambda)x_2) > \lambda f(x_1) + (1 - \lambda)f(x_2)$, for all $x_1, x_2$ and all $\lambda$ in $(0,1)$. 

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2.2 Legislative Organization Under a Closed Rule

A legislature \( L \) must select a vector of projects, \( X = \{x_1, x_2, x_3, ..., x_n\} \). The game is an infinite-horizon bargaining model with the following structure. At the beginning of every period, nature selects a legislator at random to serve as an agenda setter. The agenda setter makes a proposal which consists of a proposed project for each district, with a possible project size being zero. Let \( x \) represent the size of the proposer’s project, and \( y \) the size of projects for those members who receive an offer from the proposer. (By symmetry, in equilibrium \( y \) will be the same for all members who receive a project.) The legislature, operating under a closed rule (i.e., no amendments allowed), then votes on the proposal by \( s \)-majority rule, \( s \geq \frac{n+1}{2} \). To eliminate open set problems, a legislator who is indifferent between voting for or against a proposal is assumed to vote for it. To eliminate equilibria that involve legislators voting against legislation that gives them higher utility than the alternative, weakly dominated strategies are also ruled out. If the proposal is accepted, the game ends. If the proposal is rejected, nature selects a new agenda setter at random, and she offers a new proposal. The game continues indefinitely until an agreement is reached. This game has an infinite number of Nash equilibria; I focus on a subset of these. The equilibrium concept is subgame perfect Nash restricted to the consideration of stationary strategies, in which players must take the same actions at every node in which the game is structurally identical.\(^6\) This means that in every period, the same equilibrium offers will be made. Delay in bargaining is accounted for by a discount factor, \( \delta \in (0, 1] \), with a payoff in period \( t \), \( t \in [1, 2, ..., \infty) \), being discounted by \( \delta^{t-1} \). A representation of the game’s extensive form appears in Figure 1.

\(^6\)See Baron and Kalai (1993) for a discussion of the “focal quality” of the stationary equilibrium. They show that the stationary equilibrium is the “simplest” equilibrium of a Baron-Ferejohn game with infinitely-many subgame perfect Nash equilibria.
2.3 Legislative Organization Under an Open Rule

A legislature $L$ must select a vector of projects, $X = \{x_1, x_2, x_3, ..., x_n\}$. The game is an infinite-horizon bargaining model with the following structure. Nature selects a legislator at random to serve as an agenda setter. The agenda setter makes a proposal. As before, let $x$ represent the size of the proposer’s project, and $y$ the size of projects for those members who receive an offer from the proposer. To eliminate open set problems, a legislator who is indifferent between voting for or against a proposal is assumed to vote for it. To eliminate equilibria that involve legislators voting against legislation that they prefer to the alternative, weakly dominated strategies are also ruled out. If the proposal is accepted, the game ends. After the agenda setter proposes legislation, a legislator is selected randomly to either propose an amendment to the legislation, or to move the previous question (MPQ). If he moves the previous question, then the previous question (PQ) comes up for a vote under $s$-majority rule, $s \geq \frac{n+1}{2}$. If it is accepted, the game ends. If it is rejected, a new agenda setter is chosen and the game begins again. If he proposes an amendment, then the proposal and the amendment are pitted against each other. The proposal receiving the most votes is then subject to amendment, as before. The game continues indefinitely until an agreement is reached. Delay in bargaining is accounted for by a discount factor, $\delta \in (0, 1]$, with a payoff in period $t \in [1, 2, ..., \infty)$ discounted by $\delta^{t-1}$. Let $k \geq s$ be the number of members that receive projects in equilibrium. Unlike under a closed rule, this may be larger than a minimum winning coalition, because the agenda setter may want to insure against amendments being made to his proposal. A representation of the game’s extensive form appears in Figure 2.

3 Comparing Open and Closed Rule Outcomes

The technical solutions to these models do not provide intuitive results, so formal statements of the equilibria and proofs appear in the appendix. The following features are of interest when comparing open and closed rules: (a) the size (and efficiency) of projects received by
the agenda setter and by coalition members; (b) the expected value of the game; (c) net
benefits for players in the model; and (d) total spending.

Under a closed rule, the agenda setter gains at the expense of other players and secures
an extremely inefficient project for himself, while giving other players efficient or slightly
inefficient projects (depending on their patience). The agenda setter can exploit the other
members of the coalition because they have no power to amend his proposal and either
must accept it or reject it and move to the next period, where they may face an even
worse outcome: receiving no project at all. The agenda setter has a huge advantage in this
model, then, and this leads to a collectively inefficient outcome. The agenda setter’s power
is constrained under an open rule because the amender is sure to offer an amendment if he or
she dislikes the agenda setter’s proposal. This forces him to propose a much smaller project
for himself that leads to massive efficiency gains.

We can vary key parameters of the model—s, n, and δ—for particular values of b, c, and
n. Consider the case of b = c = 1 for n = 435 and n = 101, the size of the U.S. House
of Representatives and the U.S. Senate (plus one for convenience), respectively. Table 1
demonstrates how equilibrium outcomes change as δ, s, and n are varied. These relationships
hold:7

1. The agenda setter’s project size and net benefits are larger under a closed rule than
under an open rule.

2. All other legislators have higher net benefits and weakly larger projects under an open
rule than under a closed rule.

3. The expected value of the closed rule is strictly lower than that of an open rule, except
when legislators are perfectly patient and the voting rule is unanimity.

4. Spending is lower under an open rule than under a closed rule.

7In future iterations of this paper, I intend to solve for the conditions under which these relationships
hold. Based on my research to date, I conjecture that they hold for most parameter values.
5. Project sizes are almost always inefficient under both closed and open rules, but that inefficiency tends to be larger under a closed rule for the agenda setter’s project and larger for the coalition members under an open rule.

6. The larger the coalition required for bill passage, the better off the legislature, regardless of the amendment rule.

These results follow from the different bargaining logic under closed and open rules. The closed rule in this model will tend to give an enormous advantage to the agenda setter, just as in a divide-the-dollar framework. The difference in this case is that the size of spending is not prefixed. The closed rule gives the agenda setter the ability to select an extremely inefficient project for himself. He is more constrained under an open rule because amendments can nullify his proposal before it is voted on directly. In addition, under an open rule, the agenda setter has to offer better projects to members of his coalition so that they do not have any incentive to offer amendments, and his project must be less inefficient. Under a closed rule, legislators will accept smaller offers because they do not know who will be selected to be the next agenda setter if the game continues. Under an open rule, once a legislator is selected to make an amendment or move the previous question, he is in a stronger position, since he knows he can make an amendment and therefore needs to be “paid” more.

The expected value of the game is greater and spending lower under an open rule, compared to a closed rule. The major impact on spending comes from the dramatic reduction in the agenda-setter’s project under an open rule. While delay is possible under an open rule in equilibrium, the potential welfare losses from delay are outweighed by the efficiency gains due to changes in the agenda setter’s project. Put differently, the expected value is lower under a closed rule because the benefit of eliminating delay risk is outweighed by the cost of the oversized project the agenda setter will receive. The above results follow from this logic.

There are two important implications of this model: First, open rules tend to be preferred by the legislature to closed rules. This result differs from Baron and Ferejohn’s (1989) finding
in a distributive politics model that closed rules are preferred. Second, the agenda setter’s preference is for a closed rule.

Still, inefficiencies are present under an open rule, and the legislature may wish to address this issue by trying to impose spending limits on the agenda setter. These results suggest that it may be possible for the legislature to offer the agenda setter a closed rule on his legislation if he abides by a spending limit. The key to the successful enforcement of this provision will be that the additional benefits to the legislature obtained by limiting spending outweigh the loss from switching to a closed rule.

4 Spending Limits and the Enforcement Problem

Neoclassical economics and much institutional analysis in political science takes as a given that contracts and rules are perfectly (and costlessly) enforceable. The reality is much more complex, of course, which has prompted interest in rule enforcement and the selection of institutions. The problem of enforceability is particularly acute in the area of budgetary politics. Whether there is an external arbiter (as in a state Supreme Court, in the case of constitutional balanced budget rules, or the European Council in the case of the European Union’s Stability and Growth Pact) or rules must be enforced within the institution (as in the U.S. budget reconciliation process), the ex post incentives to violate rules is often great. The success of a rule, therefore, will depend on how well it is enforced.

Bohn and Inman (1996) have demonstrated that states with elected Supreme Courts are better able to enforce their balanced budget rules than states with appointed Supreme Courts. Elsewhere I show both formally and empirically that externally enforceable spending limitations tend to lower spending (Primo 2003a,c). But there is no a priori reason for expecting that external enforcement will be more effective than internal enforcement, since examples of ineffective enforcement come in both flavors.

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8See Baron (1991) for a different setup that also suggests a preference for open rules under certain conditions.
In 1997, European Union countries agreed to the Stability and Growth Pact, which required that EU members keep budget deficits limited to three percent of GDP or face the potential for penalties of up to half a percent of GDP. In addition, 2006 was the target for the elimination of deficits. In 2002 and 2003 the Stability and Growth Pact unraveled, with the EU Council demonstrating an unwillingness to enforce the pact and countries such as Germany conceding that budgets would not be balanced by 2006. The failure of this budget rule to bind, even with an outside enforcer (the Council), demonstrates the difficulty of designing effective budget rules.

Another example of enforcement failure is Gramm-Rudman-Hollings (GRH) legislation from the 1980s. GRH set deficit targets that were to be met by sequestration of funds (i.e., spending cuts) if those targets were not met. The goal was to eliminate the deficit by 1991, with a set of targets for each year, beginning in fiscal year 1986; the target date was later extended to 1993. This rule failed because the targets were too ambitious. At one point GRH authorized the Office of Management and Budget to make cuts of 30% in some areas, which is not politically palatable (U.S. Senate 1998, 10). The Congress subsequently altered the law.

These examples are meant to illustrate that enforcement of budget rules is particularly challenging. A need for rules, of course, implies the existence of a problem that the rules are intended to solve. One potential concern, as we just observed, is the power of the agenda setter. While I have demonstrated that budget caps can be effective in the U.S. states when exogenously enforced, this leads naturally to a new question: when exogenous enforcement is not feasible, is endogenous enforcement of spending limitations possible in a distributive politics framework? Specifically, is endogenous enforcement of a budget limit feasible in the context of a distributive politics model where spending is not pre-fixed?

Instead of taking the amendment rules as given, the endogenous enforcement model ties the amendment rule to the agenda setter’s adherence to a preset spending limit. A representation of the game’s extensive form appears in Figure 3. At the beginning of the
game, the legislature sets a limit on total spending. A rule is attached to this limit which states that any legislation satisfying this cap will be considered on the floor under a closed rule, and any legislation violating this cap will considered on the floor under an open rule. Because all legislators are identical and have equal opportunities of being selected to serve as the agenda setter, the first period decision will be unanimous, and the collective choice mechanism for this period does not need to be specified. Next, the agenda setter makes a proposal. If the total costs of the proposal are within the spending limit proposed by the legislature, then the chamber operates under a closed rule (i.e., no amendments allowed) and votes on the proposal by \( s \)-majority rule, where \( s \in \left[ \frac{n+1}{2}, n \right] \) is the number of legislators required to enact a piece of legislation. If the proposal is rejected, the legislature reconvenes to select a new spending limit, and the game continues. If the initial proposal violates the limit set by the legislature, then the chamber operates under an open rule as defined earlier.

This structure makes use of the fact that the agenda setter will tend to be hurt by the open rule, since he must build a coalition to take into account a new proposer who can choose to amend the legislation. There are two key choices in this game: First, the agenda setter must decide, as a function of the spending limit, whether to pursue an open or a closed rule strategy. Second, the legislature, knowing the agenda setter’s equilibrium strategy as a function of budget size, must select the budget size appropriately.

### 5 Successful Enforcement?

By stationarity, the game can be separated out into closed and open rule versions. The closed rule model can be solved as a function of budget size. (This is solved as Proposition 3 in the appendix.) For every budget size, then, the agenda setter will choose to stay within the limit only if the value of the game under that limit and a closed rule is higher than under an open rule and an unlimited budget. Then the legislature’s choice is simple. The legislature will select the budget size that provides the maximum expected net benefits to
the legislature.

In general, the budget cap has the effect of lowering the size of overall spending and increasing the expected net benefits of the game, relative to the baseline closed rule model. The main reason is that in a closed rule model without a spending limit, the agenda setter receives a project that is significantly larger than all other projects. By instituting a cap, the legislature can structure the agenda setter’s problem such that the expected net benefits of the game will be positive.9

Note that in a stationary equilibrium of the complete model, if an agenda setter adheres to a closed rule in one period, he must do so in every period, and similarly, the legislature must select the same budget limitation every time it sets a limitation. Therefore, to see what the legislature should do, the expected net benefits of the open rule model are compared with the expected net benefits of the closed rule model with a budget cap.

The equilibrium to this game varies depending on the values of $b$, $c$, $n$, $s$, and $\delta$. For cases where the legislature wishes to impose a closed rule with a cap, the equilibrium is as follows: In the first period, the legislature proposes a budget cap where spending maximizes expected net benefits. The players then follow the equilibrium strategy of the closed rule model with a spending limit. When an open rule is preferable, then the legislature will choose not to set a limit in the first period, and then in subsequent periods, players follow the equilibrium strategy described in the open rule result.

We cannot establish precise conditions for when a budget cap will be imposed successfully. However, from a series of simulations varying the parameters of the model, it becomes clear that as the size of the coalition required to pass legislation increases, the desire for an open rule increases. The same relationship is present as $\delta$ increases, though this effect is not large. Intuitively, as $s$ increases, less players are being left out of the coalition, and the distributive politics problem partially resolves itself.10 This makes setting a budget cap less attractive.

9When solving the complete model, note that if the legislature can put into place this equilibrium, then we need not check any other levels of spending, since this is the best the legislature can do when a budget cap is in place with a closed rule.

10For sufficiently high $n$, the agenda setter always builds minimum-winning coalitions (i.e., $k^* = s$).
One drawback to an open rule is that delay occurs with probability \( \frac{n-k}{n-1} \), or the probability that a member not given a project is selected to make an amendment. As \( \delta \) increases, delay is less costly, therefore making the open rule more attractive.

As an example, let \( b = c = 1 \), \( n = 435 \), and let \( s \) vary from 218 to 435. Figure 4 presents the expected net benefits of the open rule model and the closed rule budget cap model, for \( \delta = .4, .6, .8 \), and 1. The agenda setter’s net benefits are always greater under a closed rule with a budget cap than under an open rule, so the only question that remains is when the legislature’s welfare will be larger under a closed rule with a budget cap than under an open rule. These graphs demonstrate that for most values of \( s \) and \( \delta \), an open rule will be selected and endogenous enforcement of budget caps will not be achieved. And as \( \delta \) increases, the value of \( s \) where the legislature begins to prefer an open rule to a closed rule gets smaller. So for example, when \( \delta = .4 \), the legislature prefers an open rule only for a coalition size greater than or equal to 55. By comparison, for \( \delta = 1 \), the legislature prefers an open rule for all coalition sizes except a bare majority. In short, then, endogenous enforcement is far from being assured through the use of amendment rules. Success depends on the size of the majority required for legislation to pass, and endogenous enforcement of a budget limit through the use of amendment rules will tend to fail, except for simple majority rule. This reinforces why open rules are so prevalent on appropriations legislation. A legislature’s ability to limit spending depends on the tradeoff between reducing spending and thereby reducing the tendency of the agenda setter to propose inefficient projects, and the equalizing effect of the open rule on the projects received by the agenda setter and other members. Neither effect dominates in all cases, but the open rule tends to be preferred.

6 Discussion

This paper makes two contributions to the literature on bargaining and legislative organization. First, a distributive politics model demonstrates that open rules will be preferred
to closed rules by a legislature when spending is not prefixed. This differs from Baron and Ferejohn’s divide-the-dollar finding and is consistent with the U.S. House of Representatives and U.S. state budget processes. Second, this preference for open rules is so strong that only in limited cases is the legislature willing to trade-off an open rule for a promise by the agenda setter to adhere to a spending limit. In cases where spending limits cannot be enforced outside the legislature, this suggests that it will be difficult for legislatures to enforce them within the legislature through the use of amendment rules.

In future work I will consider what other alternatives exist for the endogenous enforcement of budget rules. Ultimately, the success of rule enforcement will rely on how well those rules match the institutional framework in which they operate and the incentives of participants. The view of institutions used in this paper should be a fruitful avenue for further work. One natural extension of this paper is to examine the institutional designs most amenable to the internal enforcement of rules. This admits a combination of game theoretic and empirical analysis, and it where this project turns next.
A Appendix

Proposition 1 (Closed Rule Model) The subgame perfect Nash equilibrium in stationary strategies is as follows. Define $\lambda^* = \frac{2\delta(s-1) - n + \sqrt{(2\delta(s-1) - n)^2 - (s-1)(2\delta + (1-\delta)n)(n(\delta+1) - 2\delta(s-1))}}{n + \delta n - 2\delta(s-1)}$. In every period, the agenda setter proposes $y^* = \frac{\lambda^* b n}{(1+\lambda^*)(s-1)c}$ to $s-1$ legislators and $x^* = \frac{b n}{(1+\lambda^*)c}$. In every period, those members who receive an offer of at least $y^*$ vote for it, and all other legislators vote against it. The agenda setter accepts offers of at least $y^*$, and since $x^* > y^*$, the agenda setter votes for the proposal.

Proof of Proposition 1: Closed Rule Model

The agenda setter chooses $x$ and $y$ to maximize

\[ bx - \frac{c}{2n}(x^2 + (s-1)y^2) \]

s.t. $by - \frac{c}{2n}(x^2 + (s-1)y^2) - \delta v \geq 0$

where $v$ is the equilibrium continuation value of a legislator receiving an offer of $y$. In addition,

\[ v = \frac{bx^*}{n} + \frac{(s-1)by^*}{n} - \frac{c(x^*^2 + (s-1)y^*^2)}{2n} \]

The Lagrangian is

\[ \mathcal{L} = bx - \frac{c}{2n}(x^2 + (s-1)y^2) + \lambda(by - \frac{c}{2n}(x^2 + (s-1)y^2) - \delta v) \]

which gives three first-order conditions:

\[ \frac{\partial \mathcal{L}}{\partial x} = b - (1 + \lambda)\frac{cx^*}{n} = 0 \]

\[ \frac{\partial \mathcal{L}}{\partial y} = -(1 + \lambda)\frac{(s-1)cy^*}{n} + \lambda b = 0 \]

\[ \frac{\partial \mathcal{L}}{\partial \lambda} = by^* - \frac{c}{2n}(x^*^2 + (s-1)y^*^2) - \delta v = 0 \]
Solving for $x^*$ and $y^*$ in terms of $\lambda^*$ gives

$$
x^* = \frac{bn}{(1 + \lambda^*)c} \quad \text{and} \quad y^* = \frac{\lambda^*bn}{(1 + \lambda^*)(s - 1)c}.
$$

The third first-order condition and the definition of $v$ gives

$$
2\delta bx^* + (1 - \delta)cx^2 = 2by^*[n - \delta(s - 1)] + c(s - 1)y^2(\delta - 1).
$$

(1)

Substituting the relations for $x$ and $y$ into (1) implies that

$$
\lambda^* = \frac{2\delta(s - 1) - n + \sqrt{(2\delta(s - 1) - n)^2 - (s - 1)(2\delta + (1 - \delta)n)(n(\delta + 1) - 2\delta(s - 1))}}{n + \delta n - 2\delta(s - 1)}.
$$

This completes the statement of the equilibrium. Finally, it can easily be verified that neither defection nor building a larger than minimum-winning coalition is ever optimal.

\[\square\]

**Proposition 2 (Open Rule Model)** The following characterizes one of the subgame perfect Nash equilibria in stationary strategies. In the first period, the agenda setter chooses $(k^*, x^*, y^*)$ that maximizes

$$
\frac{k - 1}{n - 1}(bx - \frac{c}{2n}(x^2 + (k - 1)y^2)) + \delta v_1(z^m)
$$

s.t. $by - \frac{c}{2n}(x^2 + (k - 1)y^2) - \delta v_1(z^1) \geq 0,$

where $v_1^*(z^1)$ and $v_1^*(z^m)$ are continuation values determined in equilibrium. The player selected to make an amendment then moves the previous question if he was offered $y^*$ by the agenda setter, and the bill passes. If he was not offered a project, he makes an amendment where he offers $y^*$ to the $(n - k)$ members who did not receive a project initially, and then allocates the other $(2k - n - 1)$ projects of size $y^*$ randomly to the other members, except for the first agenda setter. He offers himself a project of size $x^*$. Then another player is selected to make an amendment or to move the previous question, and he follows the same strategy given above. Legislators vote for any bill giving them a project of at least $y^*$. Project sizes are $x^* = \frac{bn(k^*-1)}{c((k^*-1)+\lambda^*(n-1))}$ and $y^* = \frac{bn(n-1)\lambda}{c(k^*-1)((k^*-1)+\lambda^*(n-1))}$, where

$$
\lambda^* = \frac{(k-1)2\delta v_1^*(z^1)ck^*-b^2n-2\delta v_1^*(z^1)c+\sqrt{b^2n(b^2n-2\delta v_1^*(z^1)ck^*+2\delta v_1^*(z^1)c)}}{2\delta v_1^*(z^1)c-2\delta v_1^*(z^1)ck^*+b^2n},
$$

$v_1^*(z^1)$ is defined in the proof, and $k^*$ is the coalition size that maximizes the agenda setter’s expected utility (typically the minimum winning coalition).
Proof of Proposition 2: Open Rule Model

This proof has elements that are similar to the Baron and Ferejohn (1989) proof of the open rule model. An additional assumption, left out of their proof, is made here: when building a coalition, an amender must always “buy” members without a project in the existing bill before buying others.\(^{11}\) Also, for simplicity, let \(NB(\cdot)\) denote the net benefits to a player receiving an equilibrium project, along with the costs associated with that project and all other projects in the bill. Finally, assume that a legislator votes for the most recently proposed bill when indifferent between two pieces of legislation.

Let’s define some parameters to make the presentation clearer. Let \(\alpha = (1 - \delta(1 - \frac{k-1}{n-1})(\frac{2k-n}{k-1}))^{-1}\). Let \(\beta = (1 - \alpha\delta^2(1 - \frac{k-1}{n-1})(\frac{n-k-1}{k-1})(1 - \frac{k-n}{n-1}))^{-1}\). Let \(\psi = (1 - \delta^2(1 - \frac{k-1}{n-1})(\frac{1}{n-1}) - \alpha\beta\delta^4(1 - \frac{k-1}{n-1})^2(\frac{2k-n}{k-1})^2(\frac{k-1}{k-1})^2(\frac{n-k-1}{n-1})^2(1 - \frac{k-n}{n-1}))^{-1}\). \(k\) is defined below.

Before solving the agenda-setter’s maximization problem, the continuation values for various players in the model are calculated. Let \(v_i(z^j)\) be the value to player \(i\) of an offer \(z\) by a player \(j\), where \(z\) is shorthand for the set of projects offered by a player. Denote the first player chosen as the agenda setter “player 1.” Then \(v_1(z^1)\) is the value of the game to player 1 when he has the ability to make an offer.

I proceed by constructing \(v_1(z^1)\). To forestall an amendment, the agenda setter must provide a project large enough that the player receiving the project has no incentive to amend the bill. Thus he must pay these players, call a single one \(p\), \(\delta v_1(z^1)\). (Players receiving no project will propose an amendment. The size of the agenda setter’s coalition is determined in equilibrium.) Let \(k^* \geq s\) be the number of players receiving a project in the equilibrium, including the agenda setter.

This implies that
\[
v_1(z^1) = \frac{k^* - 1}{n-1} NB(x^*) + (1 - \frac{k^* - 1}{n-1})\delta v_1(z^m),
\]
where \((1 - \frac{k^* - 1}{n-1})\) reflects the probability that a player \(m\) not offered a project is selected to amend the agenda setter’s bill, in which case the agenda setter expects to receive \(v_1(z^m)\) in

\(^{11}\)See Primo (2003b) for a note on coalition building in distributive politics games under an open rule.
the next period, therefore causing a one-period discount. The agenda setter here is in the
same position as was at the beginning of the game, so \( v_1(z^m) = v_m(z^1) \).

Next,
\[
v_m(z^1) = \frac{k^* - 1}{n-1} NB(0) + \frac{\delta}{n-1} v_1(z^1) + (1 - \frac{k^*}{n-1}) \delta v_p(z^1).
\]

This follows from the fact that \( m \) receives no project if one of the \( p \) players moves the
previous question. He has a small chance of being selected to make an amendment, therefore
putting him in the same situation as the present agenda setter, in which case he receives
\( v_1(z^1) \). Finally, he has a \( (1 - \frac{k^*}{n-1}) \) chance that another player lacking a project will be
chosen to be the agenda setter, which guarantees that he’ll be selected to get a project, by
the assumption about coalition building given at the beginning of the proof, thereby giving
him a continuation value similar to a player offered a project in period 1.

Similarly,
\[
v_p(z^1) = \frac{k^* - 1}{n-1} NB(y^*) + \delta (1 - \frac{k^* - 1}{n-1}) v_p(z^m).
\]

Next,
\[
v_p(z^m) = \frac{2k^* - n}{k^* - 1} \left( \frac{k^* - 1}{n-1} v_1(z^1) + \delta (1 - \frac{k^* - 1}{n-1}) v_p(z^m) \right)
+ \frac{n - k^* - 1}{k^* - 1} \left( \frac{1}{n-1} v_1(z^1) + \delta (1 - \frac{k^*}{n-1}) v_p(z^1) \right) + \frac{k^* - 1}{n-1} NB(0)
\]

This follows from the fact that player \( p \) has a \( \frac{2k^* - n}{k^* - 1} \) chance of receiving a project, in
which case he is in the same position as \( p \) was when he was offered a project by player 1. \( v_p(z^1) \). This probability comes from the rule that player \( m \) follows when building a coalition.
First he makes offers to players without projects, and then he gives the remaining projects
to players who received projects in the earlier bill. Player \( p \) has a \( 1 - \frac{2k^* - n}{k^* - 1} \) chance of not receiving a project, in which case he receives \( v_m(z^1) \).

Next, by substituting \( v_p(z^m) \) into the expression for \( v_p(z^1) \), and then substituting \( v_p(z^1) \)
into the expression for \( v_m(z^1) \), and then finally by substituting \( v_m(z^1) \) into \( v_z(z^1) \), we obtain
\[ v_1(z^1) = \psi \left( \frac{k^* - 1}{n - 1} NB(x^*) + \beta \delta^2 (1 - \frac{k^*}{n - 1}) \left( \frac{k^* - 1}{n - 1} \right) \left( 1 - \frac{k^* - 1}{n - 1} \right) NB(y^*) \right. \\
+ \delta (1 - \frac{k^*}{n - 1}) \left( \frac{k^* - 1}{n - 1} \right) NB(0) \\
\left. + \delta^3 \alpha \beta (1 - \frac{k^*}{n - 1})^2 \left( 1 - \frac{k^*}{n - 1} \right) \left( \frac{n - k^* - 1}{n - 1} \right) \left( \frac{n - k^* - 1}{k^* - 1} \right) NB(0) \right) \]

or

\[ v_1(z^1) = \psi \left( \frac{k^* - 1}{n - 1} \left( bx - \frac{c}{2n} (x^* + (k^* - 1)y^*) \right) \right. \\
+ \beta \delta^2 (1 - \frac{k^*}{n - 1}) \left( \frac{k^* - 1}{n - 1} \right) \left( 1 - \frac{k^* - 1}{n - 1} \right) \left( by - \frac{c}{2n} (x^* + (k^* - 1)y^*) \right) \\
+ \delta (1 - \frac{k^*}{n - 1}) \left( \frac{k^* - 1}{n - 1} \right) \left( - \frac{c}{2n} (x^* + (k^* - 1)y^*) \right) \\
\left. + \delta^3 \alpha \beta (1 - \frac{k^*}{n - 1})^2 \left( 1 - \frac{k^*}{n - 1} \right) \left( \frac{n - k^* - 1}{n - 1} \right) \left( \frac{n - k^* - 1}{k^* - 1} \right) \left( - \frac{c}{2n} (x^* + (k^* - 1)y^*) \right) \right). \]

The agenda setter picks \((x, y)\) that maximizes

\[ \frac{k - 1}{n - 1} \left( bx - \frac{c}{2n} (x^2 + (k - 1)y^2) \right) + \delta v_1(z^m) \text{ s.t. } by - \frac{c}{2n} (x^2 + (k - 1)y^2) - \delta v_1(z^1) \geq 0 \]

The Lagrangian is

\[ \mathcal{L} = \frac{k - 1}{n - 1} \left( by - \frac{c}{2n} (x^2 + (k - 1)y^2) \right) + \delta v_1(z^m) + \lambda \left( by - \frac{c}{2n} (x^2 + (k - 1)y^2) - \delta v_1(z^1) \right) \]

which gives three first-order conditions:

\[ \frac{\partial \mathcal{L}}{\partial x} = \frac{k - 1}{n - 1} b - \frac{c}{n} \left( \frac{k - 1}{n - 1} + \lambda \right) x = 0 \]

\[ \frac{\partial \mathcal{L}}{\partial y} = \lambda b - \frac{(k - 1)c}{(n - 1)n} \left( \lambda (n - 1) + (k - 1) \right) y = 0 \]

\[ \frac{\partial \mathcal{L}}{\partial \lambda} = \left( by - \frac{c}{2n} (x^2 + (s - 1)y^2) - \delta v_1(z^1) \right) = 0 \]

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Rearranging terms in the first two first-order conditions gives

\[ x^* = \frac{bn}{c} \frac{k - 1}{(k - 1) + \lambda(n - 1)} \]

and

\[ y^* = \frac{bn}{c} \frac{n - 1}{k - 1} \frac{\lambda}{(k - 1) + \lambda(n - 1)} \]

Equilibrium values of \( x \) and \( y \) can be determined by substituting the equations for \( x^* \), \( y^* \), and \( v_1(z^1) \) into the third first-order condition and finding \( \lambda^* \) that satisfies the constraint, for a given \( k \).

The agenda-setter also must determine the size of the coalition to build. As opposed to the closed rule case, where it is straightforward that a minimum-winning coalition is optimal, there may be reason to build oversized coalitions in the open rule case to forestall amendments.

Because \( k \) is integer-valued and must be in \([s,n]\), the equilibrium value of \( k \) can be determined as follows. To verify that a coalition of size \( k_0 \) forms an equilibrium, solve the above maximization problem for \( k = k_0 \) and note the continuation values. Then, plug those continuation values into the maximization problem. Maximize again, this time with respect to \( k \) as well as \( x \) and \( y \) (restricting \( k \) to be an integer \( \in [s,n] \)). If the maximization gives a different solution, then \( k_0 \) was not an equilibrium, since the continuation values were not consistent with utility-maximizing behavior on the part of the agenda setter. If the equilibrium values are unchanged, then \( k_0 = k^* \).

**Proposition 3 (Closed Rule Model with Budget Cap)** The subgame perfect Nash equilibrium in stationary strategies is as follows. Define

\[ \lambda = \frac{2b^2(\delta(s - 1)(n - \delta(s - 1)) + (s - 1)(1 - \delta)\sqrt{2b^2c(n - \delta(s - 1))^2I + (s - 1)2b^2\delta^2cI - c^2(\delta - 1)^2I^2}}{2b^2(n - \delta(s - 1))^2 - c(s - 1)(\delta - 1)^2I}. \]

In every period, the agenda setter proposes \( y^* = \frac{\lambda \sqrt{2I}}{\sqrt{c(s-1)(s-1+\lambda^2)}} \) to \( s - 1 \) legislators and \( x^* = \frac{\sqrt{2I(s-1)}}{\sqrt{c(s-1+\lambda^2)}} \). In every period, those members who receive an offer of at least \( y^* \) vote for it, and all other legislators vote against it. The agenda setter accepts offers of at least \( y^* \), and since \( x^* > y^* \), the agenda setter votes for the proposal. The legislature selects the value \( I^* \) that maximizes expected net benefits in the model.
Proof of Proposition 3: Closed Rule Model With Budget Cap

The agenda setter chooses $x$ and $y$ to maximize

$$bx - \frac{c}{2n}(x^2 + (s-1)y^2)$$

s.t. $by - \frac{c}{2n}(x^2 + (s-1)y^2) - \delta v \geq 0$ and

$$I - \frac{c}{2}(x^2 + (s-1)y^2) \geq 0$$

where $v$ is the equilibrium continuation value of a legislator receiving an offer of $y$, and $I$ is the budget selected by the legislature in the first period. In addition,

$$v = \frac{bx^*}{n} + \frac{(s-1)by^*}{n} - \frac{c(x^*^2 + (s-1)y^*^2)}{2n}.$$

The Lagrangian is

$$\mathcal{L} = bx - \frac{c}{2n}(x^2 + (s-1)y^2) + \lambda(by - \frac{c}{2n}(x^2 + (s-1)y^2) - \delta v) + \gamma(I - \frac{c}{2}(x^2 + (s-1)y^2))$$

which gives four first-order conditions:

$$\frac{\partial \mathcal{L}}{\partial x} = b - (1 + \lambda)\frac{cx^*}{n} - \gamma cx^* = 0$$

$$\frac{\partial \mathcal{L}}{\partial y} = -(1 + \lambda)\frac{(s-1)cy^*}{n} + \lambda b - \gamma c(s-1)y^* = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = by^* - \frac{c}{2n}(x^*^2 + (s-1)y^*^2) - \delta v = 0$$

$$\frac{\partial \mathcal{L}}{\partial \gamma} = I - \frac{c}{2}(x^*^2 + (s-1)y^*^2) = 0$$

Solving for $x^*$ and $y^*$ in terms of $\lambda^*$ and $\gamma^*$ gives
\[
x^* = \frac{bn}{(1 + \lambda^* + \gamma^* n)c} \quad \text{and} \\
y^* = \frac{\lambda^* bn}{(1 + \lambda^* + \gamma^* n)(s - 1)c}.
\]

This implies that \( y^* = \frac{\lambda^*}{s-1} x^* \).

To calculate gamma, substitute the relations for \( x^* \) and \( y^* \) into the fourth first-order condition. Algebraic simplification implies that

\[
\gamma^* = \frac{1}{n} \left[ -1 - \lambda + bn \sqrt{\frac{(s - 1) + \lambda^2}{2cI(s - 1)}} \right].
\]

Substituting this back into the relation for \( x^* \) and \( y^* \) gives the equilibrium values

\[
y^* = \frac{\lambda^* \sqrt{2I}}{\sqrt{c(s - 1)(s - 1 + \lambda^2)}} \quad \text{and} \\
x^* = \frac{\sqrt{2I(s - 1)}}{\sqrt{c(s - 1 + \lambda^2)}}.
\]

The third first-order condition and the definition of \( v \) gives

\[
2\delta bx^* + (1 - \delta)cx^* = 2b y^*[n - \delta(s - 1)] + c(s - 1)y^* (\delta - 1). \quad (2)
\]

Substituting the relations for \( x \) and \( y \) into (1) implies that

\[
\lambda^* = \frac{2b^2 \delta(s-1)(n-\delta(s-1)) + (s-1)(1-\delta)\sqrt{2b^2c(n-\delta(s-1))^2I + (s-1)2b^2\delta^2cI - c^2(\delta-1)^2I^2}}{2b^2(n-\delta(s-1))^2 - c(s-1)(\delta-1)^2I}.
\]

In the first period, the legislature selects the value of \( I \) that makes it best off, given expectations about the agenda setter’s behavior in the distributive game. Since all legislators are equally likely to be either an agenda setter or a member of the coalition receiving projects, consider the decision of a generic legislator. All legislators will vote identically.

Formally, his problem is to choose the \( I \) that maximizes
\[
\frac{b}{n} \frac{\sqrt{2I(s-1)}}{\sqrt{c(s-1+\lambda^2)}} + \frac{b(s-1)}{n} \frac{\lambda \sqrt{2I}}{\sqrt{c(s-1)(s-1+\lambda^2)}} - \frac{c}{2n} \left[ \frac{2I(s-1)}{c(s-1+\lambda^2)} + \frac{2I\lambda^2}{c(s-1+\lambda^2)} \right]
\]

Note that \(\lambda^* = f(I)\), so \(\lambda^*\) cannot be treated as a constant here. This maximization problem is not solved analytically but has well-defined properties.

Finally, it can easily be verified that neither defection nor building a larger than minimum-winning coalition is ever optimal. \(\blacksquare\)
Figure 1: Closed Rule
Figure 2: Open Rule
propose budget and rule

Figure 3: Enforcement Model
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Table 1: Equilibrium project sizes, budgets, and net benefits as a function of coalition size, legislature size, and $\delta$. The expected net benefit term refers to the \textit{ex ante} expected benefit of the game, while the other net benefit terms refer to the net benefits of the game to a legislator receiving projects of size $x^*$, $y^*$, or 0 in the equilibrium.
Figure 4: Comparison of expected net benefits in the open rule model and the closed rule budget cap model, with $b=c=1$, based on the coalition size required for legislative passage and the patience of the players. Continued on next page.
Open Rule vs. Closed Rule with Budget Cap, $\delta = .6$

Open Rule vs. Closed Rule with Budget Cap, $\delta = .4$

Legislators Required to Pass Legislation, $n=101$
Expected Net Benefits
References


